Visualizing Bivariate-Normal Distributions

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Abstract
Graphs for one variable probability, density and distribution help students understand both of the concepts and applications. Students usually have difficulties to learn multivariate random variable due to unpopularity of multivariate graphs. Some issues of using bivariate graphs are discussed with examples of using Maple to graph bivariate normal distributions.

Unlike most of one-variable distributions, a graph of the probability density function of a bivariate-normal is not easy to construct. Students may have difficulties because of the lack of visual assistance and the geometric interpretation when studying multivariate distributions. However, as the advance of the computing technology, now making a bivariate graph is not as hard as it used to be. Based on a class project used for a mathematical statistics course for the past five yeas, giving such project helps students understand the bivariate-normal distributions better with the visual support.

The project does not require students to have experience in using the computer software, such as Maple, Matlab or MathCad. This is because most of these programs have included enough self-learning materials with sufficient examples. The instructors just need provide them with one even less than one class time period tutorial of the software. Then assign students into 3-5 student groups or have them grouped as their wish. Through the discussion among the students with the suggested group size, students can share their learning how to use the software and more importantly they can also share their learning about the concept and the nature of the distributions. The project is presented as the following:

Use Maple to construct 3D graphs for the joint pdf of X and Y with the following cases.

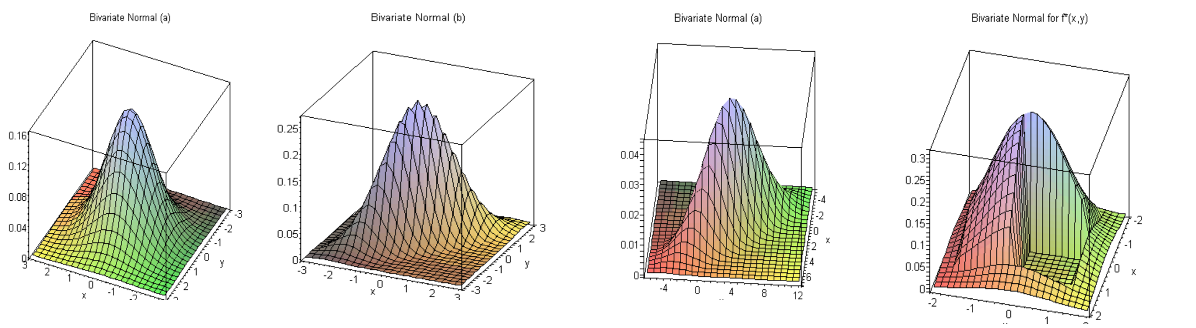
$$f(x,\ y)=\frac{e^{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{x-\mu_1}{\sigma_1}\right)^2-2\rho\left(\frac{x-\mu_1}{\sigma_1}\right)\left(\frac{y-\mu_2}{\sigma_2}\right)+\left(\frac{y-\mu_2}{\sigma_2}\right)^2\right]}}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}}$$

a. Graph $f(x,\ y)$ with $\mu_1=\mu_2=0$, $\sigma_1=\sigma_2=1$ and $\rho=0$.
b. Graph $f(x,\ y)$ with $\mu_1=\mu_2=0$, $\sigma_1=\sigma_2=1$ and $\rho=0.8$.
c. Graph $f(x,\ y)$ with $\mu_1=1$, $\mu_2=3$, $\sigma_1=2$, $\sigma_2=3$ and $\rho=-0.8$.
d. Graph $f^*(x,\ y)$ with $\mu_1=\mu_2=0$, $\sigma_1=\sigma_2=1$ and $\rho=0$ where

$$f^*(x,\ y)=\begin{cases}2f(x,y) & \text{Inside square 2 and 4 of Figure 6.10}\\ 0 & \text{Inside square 1 and 3 of Figure 6.10}\\ f(x,y) & \text{Otherwise}\end{cases}$$

Show that the marginal distribution of X and marginal distribution of Y are both standard normal.

Part a. (see graph a.) shows a standard bivariate-normal pdf with no correlation between X and Y. Part b. (see graph b.) shows what will happen when correlation is added. Part c. (see graph c.) shows the bivariate-normal pdf with different means, different standard deviations for X and Y, as well as correlation between X an Y. Part d. (see graph c.) shows how two standard normal distributions are jointed as a non-bivariate-normal distribution from which students can understand well that if joint normal then the marginal distributions must be normal, but the reverse is not true. Such important fact is little hard to learn from the proof with the definition and calculus, but the graph especially made by themselves is easy to understand and to remember.



Most of the students who worked the project feel comfortable of using Maple even they never had any experience with the software. And very surprisingly, they can come up with the same results using different way to graph part d, which is a piecewise function. For them the first experience in writing such kind of function and graph it in a computer software they never used before makes them feel confident and creative. To me it is important that they learn something in a better way and more importantly they learn how to learn.