Should Introductory Statistics Classes Include Multifactor Statistical Design of Experiments (DoE)?

USCOTS 2021

Tuesday, June 29th 3:00-3:45pm ET

David Fluharty

Adjunct Professor of Statistics and Economics

Ivy Tech Community College—Columbus IN

fluharty.earlydoe@gmail.com

Should Introductory Statistics Classes Include DoE?

- Discussion in a Beyond Session at USCOTS 2021

Tuesday, June 29th 3:00-3:45pm ET

- To Continue the Discussion and Possibly Form an "Early DoE" Community, Please Email me at

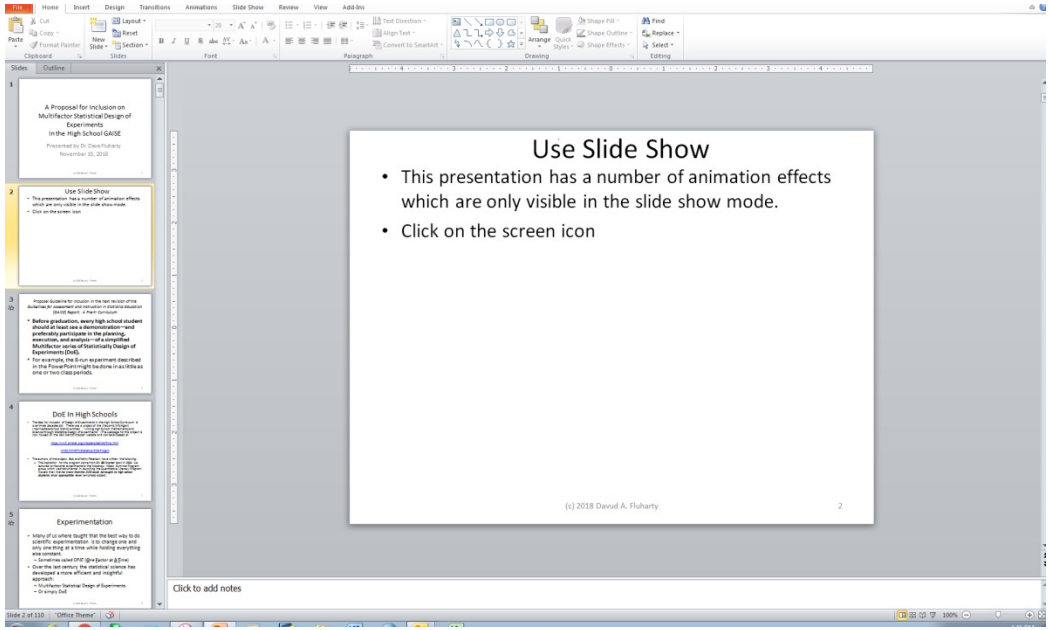
fluharty.earlydoe@gmail.com

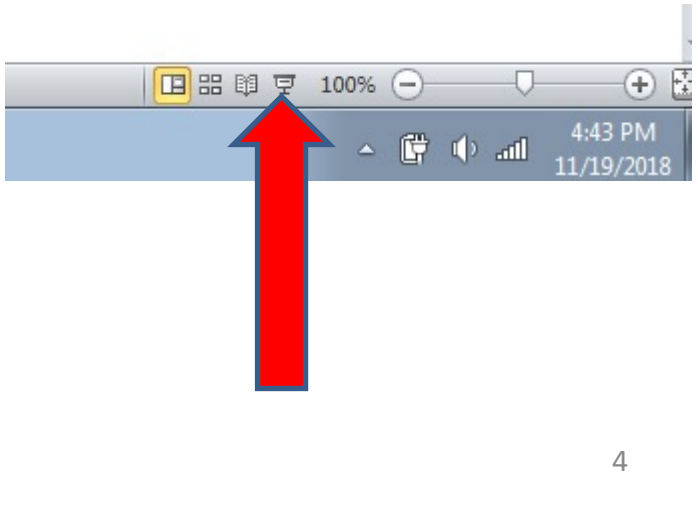
Dave Fluharty

- Adjunct Professor of Statistics and Economics, Ivy Tech Community College—Columbus Indiana
- In 2020 completed 4 decades in motor vehicle industry working in statistics, forecasting, finance, economics, reliability, and training
- PhD, Educational Evaluation and Research, Wayne State University
- MA, International Relations, University of Chicago
- MBA, Business Economics and Finance, Booth School of Business, University of Chicago

Use Slide Show Mode

- Some Slides Have Animation Effects Which Are Only Visible in Slide Show Mode.
- Click on the “Screen Icon” Near The Lower Right-Hand Corner for Slide Show Mode



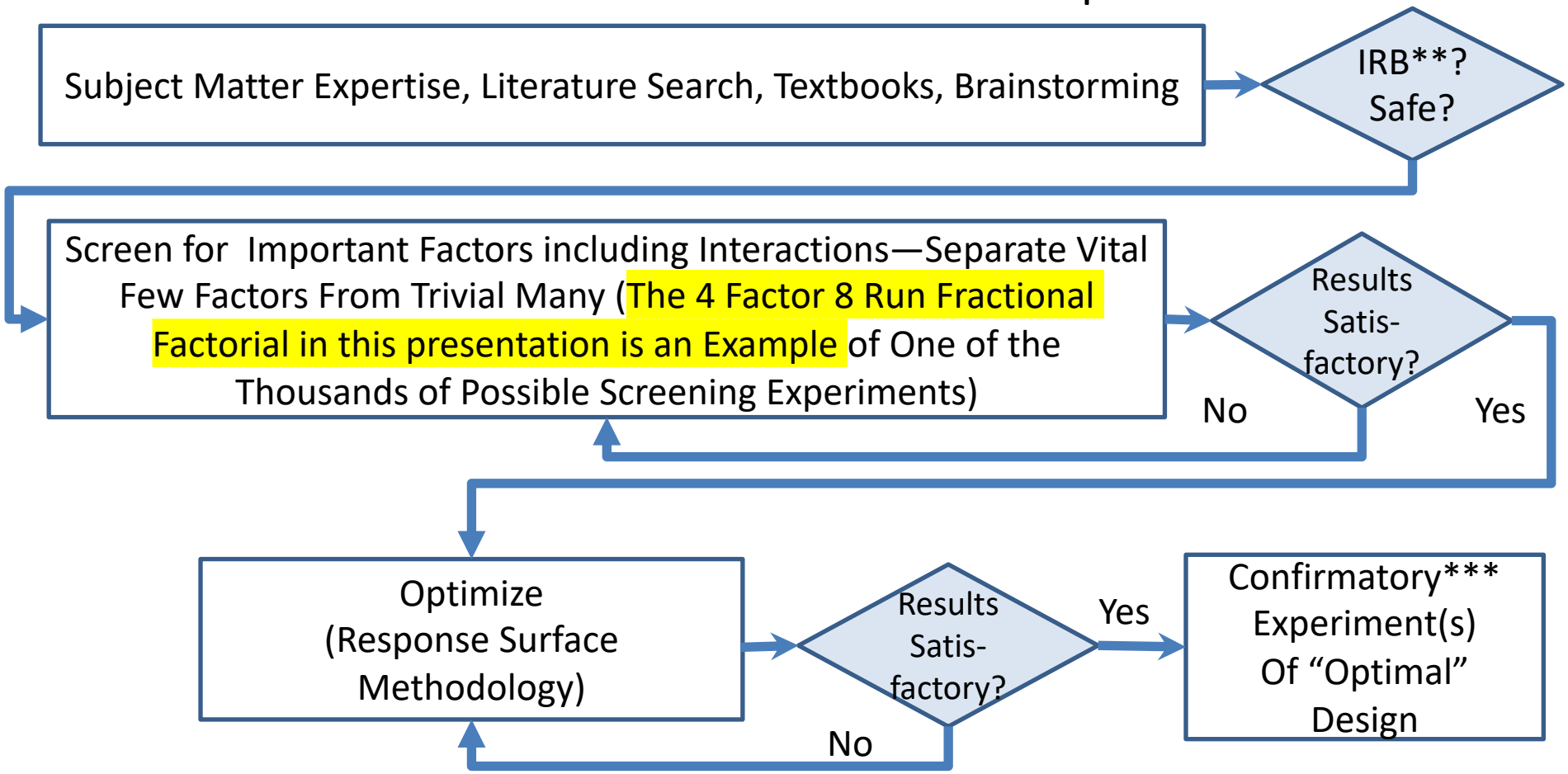


Abraham Lincoln and the Invention of Invention

- **Man is not the only animal who labors; but he is the only one who *improves* his workmanship. This improvement, he effects by *Discoveries*, and *Inventions*. . . Now, it was the destined work of Adam's race to develop, by discoveries, inventions, and improvements, the hidden treasures of this mine. But Adam had nothing to turn his attention to the work. If he should do anything in the way of invention, he had first to invent the art of invention -- the *instance* at least, if not the *habit* of observation and reflection.**

Abraham Lincoln, "Lecture on Discoveries and Inventions," Various Illinois Locations, 1858 – 1859

DoE Is an Art and Science that Helps Us Investigate the Effects of Multiple Factors (Including Interactions*) by **SIMULTANEOUSLY** Varying These Factors According to a Mathematically/Statistically **FIXED** Set of Recipes. Results Include Predictive Equations and Possibly Optimization. This Contrasts With One-Factor-At-A-Time Experimentation.

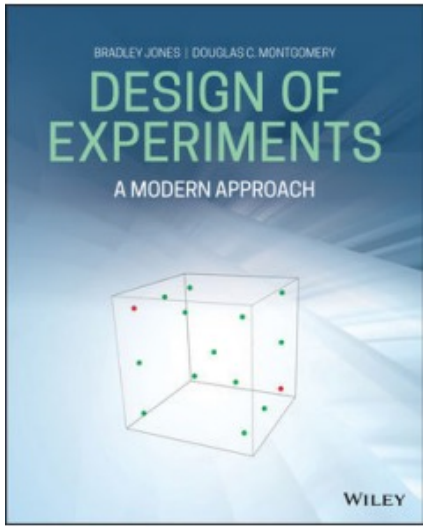


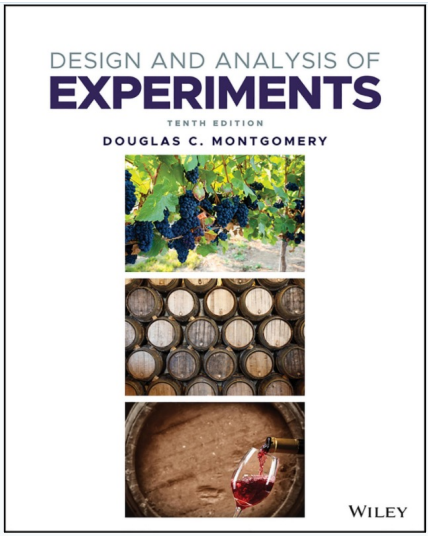
* Estimates of Interactions Depends on "Resolution" of Design

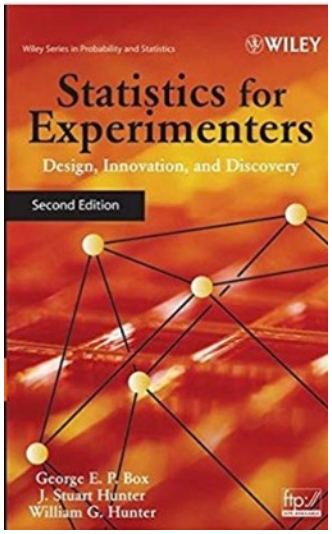
** Institutional Review Board

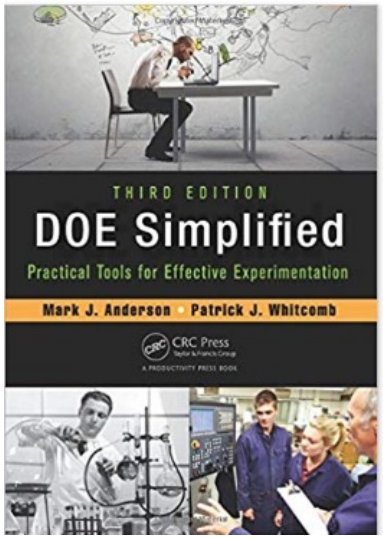
***Ensure "Optimal Result" Works as Expected (very important in industry}

Many People Who Learn DoE Learn it in a Graduate Level Course or 3-5 Day Seminar









Why Introduce DoE in Introductory Statistics?

- Why (Educational/Cognitive):
  - Multi Causal Systems (Understanding Needed in Science and Society)
  - Process of Scientific Discovery
    - Iterative
    - Discovery Vs. Demonstrative
    - Discuss Engineering Trade-Offs
  - Omnipresence of Variation
  - Interactions, Predictive Equations, and Optimization
  - Randomization
  - Statistical vs. Practical Significance (if Use Software)
  - Efficiency vs. One-Factor-At-A Time
  - Use of Statistical Graphics
  - Used Business and Industry
    - Manufacturing (Six Sigma Programs)
    - Marketing Experiments
- Pedagogy/Andragogy
  - Cognitive Domain
  - Psychomotor Domain
  - Affective Domain
    - For the Paper Helicopter Example, Model of Autorotation (Flight Time) if Engine Fails

The Flight time of 8 Different Paper* Helicopter Designs:

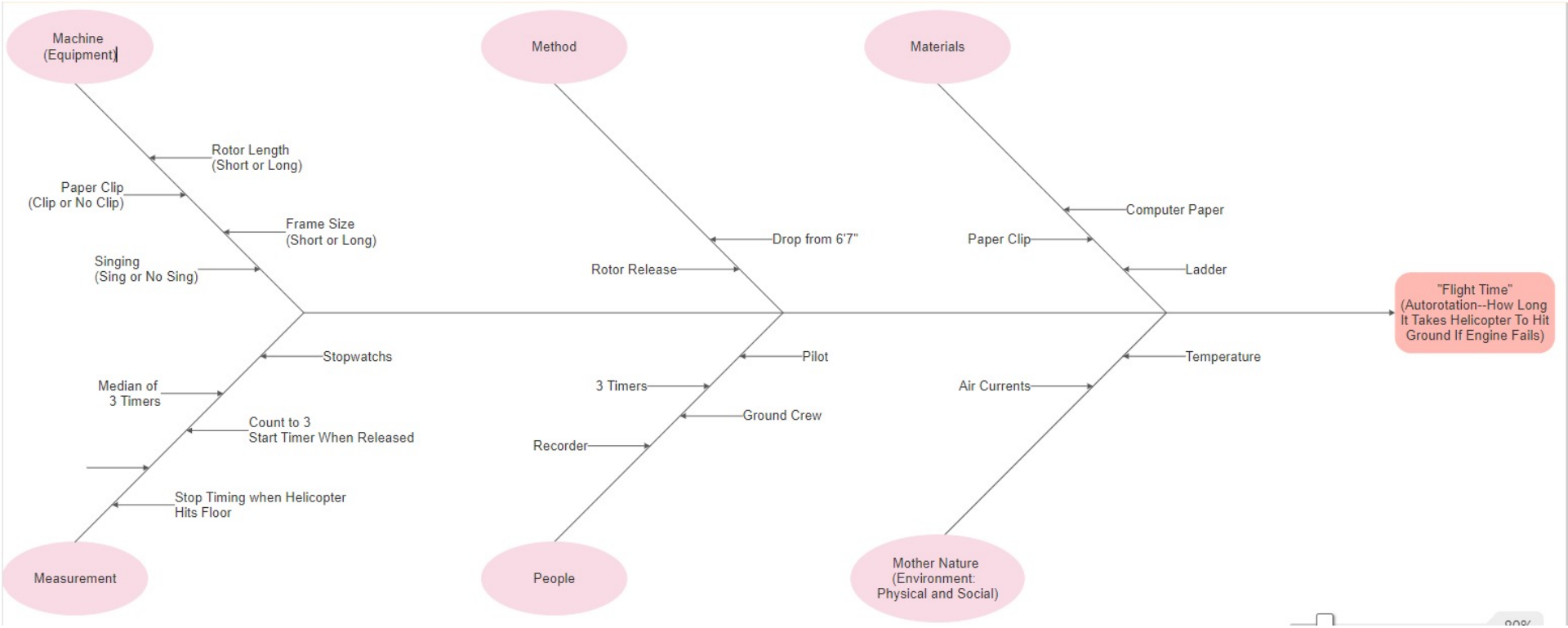
- Not Just a Toy
- A Model of ***Autorotation*** (you want this to be as long as possible—how long you stay in the air if the engine fails

Photo curtesy of Mark Anderson of Stat-Ease

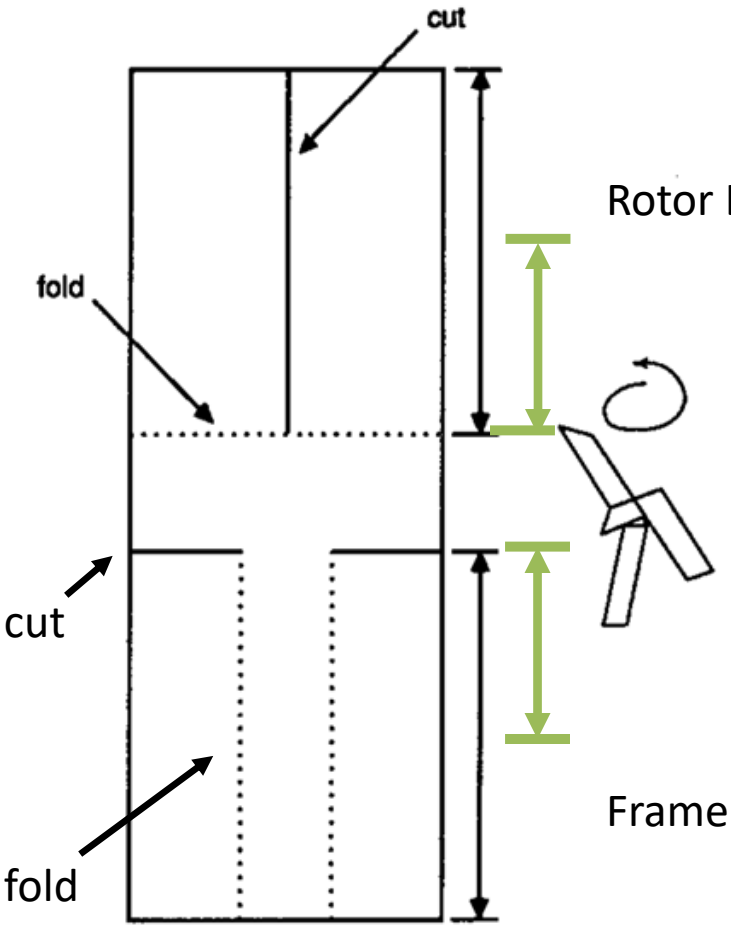
Also see https://www.statease.com/publications/newsletter/stat-teaser-03-15

*Paper Helicopter Were Popularized As a DoE Educational Tool by G.E.P.Box (Fisher's Son-In-Law)

Fishbone Diagram (Brainstorming)



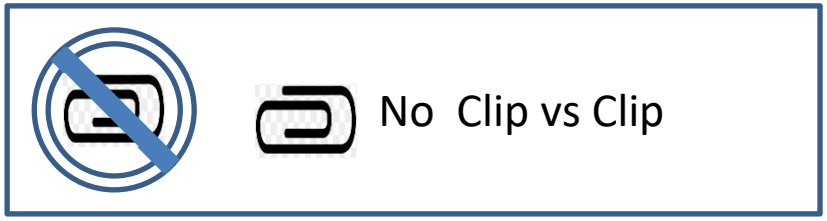
We Investigate 4 Factors That ***Might*** Impact Flight Time: Rotor Length (Long vs. Short), Frame Size (Big vs. Small), Paper Clip (No Clip vs. Clip), Singing (No Singing vs. Singing)



Rotor Length: Short and Long

No Singing vs Singing

Frame Size: Small and Big



No Clip vs Clip

Before Running the Experiments "Guesstimate" the Effects (Direction and Deviation from Average) and How Much of A Difference Matters

A Statistically Determined ***Fixed Set*** of 4 Factor 8 "Run" (Each a Different Helicopter Design) Used In This DoE Example

(There are Thousands of Other Possible Designs)

Model Name	Rotor Length	Frame Size	Clip	Singing	"Standard Order"
I	Short	Small	No Clip	No Sing	- - - -
II	Long	Small	No Clip	Sing	+ - - +
III	Short	Big	No Clip	Sing	- + - +
IV	Long	Big	No Clip	No Sing	+ + - -
V	Short	Small	Clip	Sing	- - + +
VI	Long	Small	Clip	No Sing	+ - + -
VII	Short	Big	Clip	No Sing	- + + -
VIII	Long	Big	Clip	Sing	+ + + +

Convention Used Here:

"Bigger" or "Present" is a "+"

"Smaller" or "Absent" is a "-"

Beautiful Geometry!

Why This Works—Matrix Algebra

Model Name	Rotor Length	Frame Size	Clip	Singing	"Standard Order"
I	Short	Small	No Clip	No Sing	**-1 -1 -1 -1**
II	Long	Small	No Clip	Sing	**+1 -1 -1 +1**
III	Short	Big	No Clip	Sing	**-1 +1 -1 +1**
IV	Long	Big	No Clip	No Sing	**+1 +1 -1 -1**
V	Short	Small	Clip	Sing	**-1 -1 +1 +1**
VI	Long	Small	Clip	No Sing	**+1 -1 +1 -1**
VII	Short	Big	Clip	No Sing	**-1 +1 +1 -1**
VIII	Long	Big	Clip	Sing	**+1 +1 +1 +1**

Put a "1" with every "+" and "-"
When Multiply the Rows, the Sum (Dot Product) is Zero➔
These are Orthogonal Vectors

The "Singing" Column is the product of the other three columns. For example, for Model VIII:
+1 x +1 x +1 = +1

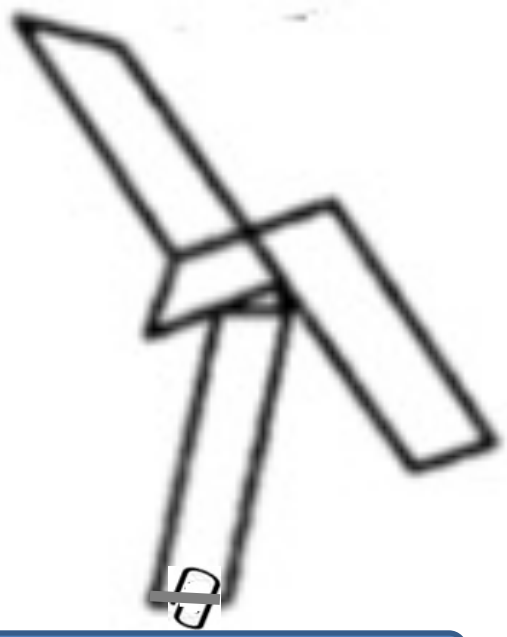
There is One Recipe Card Corresponding to Each of the Helicopter Designs: Example of Design VIII

This section contains the "recipe" for the helicopter design

The flight time of this helicopter is recorded here

The random run order is recorded here (can randomize by pulling out of a hat)

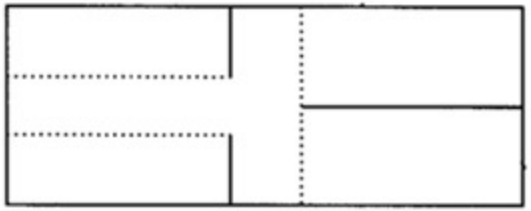
Run Order		Long Big Rotor Frame Clip Sing	Flight Time
3		+ + + +	1.62
Model	Standard		
VIII	Order	See back (unusual event)	



The model number (design name called "standard order" by statisticians

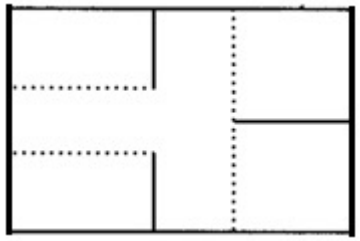
Good experimental practice is to record any unusual event during he experimental run for future analysis

Run Order		Long Big Rotor Frame Clip Sing	Flight Time
		+ + + +	
Model	Standard		
VIII	Order	See back (unusual event)	

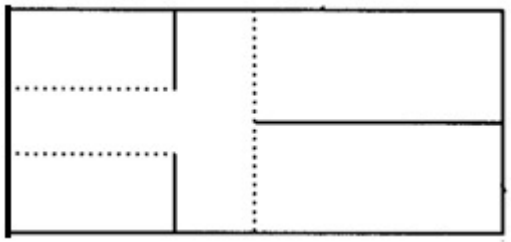


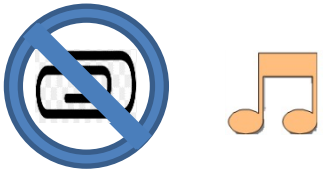
Helicopter Designs I through IV

Run Order						Flight Time
		Short Rotor	*Small Frame*	**No Clip**	*No Sing*	
Model I	Standard Order	-	-	-	-	
		See back (unusual event)				

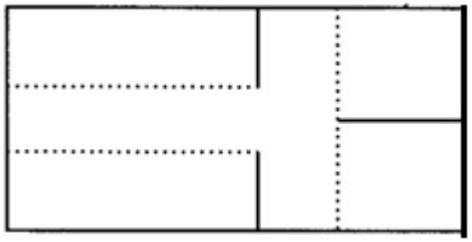


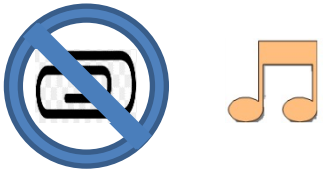
Run Order						Flight Time
		Long Rotor	*Small Frame*	**No Clip**	*Sing*	
Model II	Standard Order	+	-	-	+	
			See back (unusual event)			



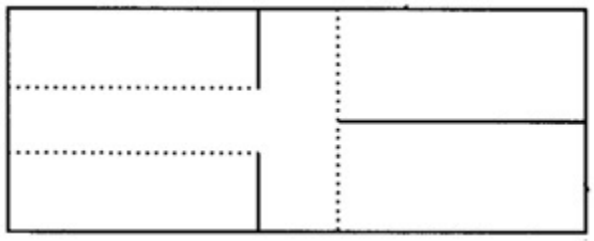


Run Order						Flight Time
		Short Rotor	*Big Frame*	**No Clip**	*Sing*	
Model III	Standard Order	-	+	-	+	
		See back (unusual event)				



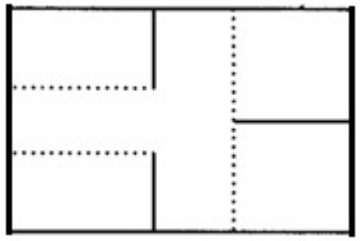


Run Order						Flight Time
		Long Rotor	*Big Frame*	**No Clip**	*No Sing*	
Model IV	Standard Order	+	+	-	-	
		See back (unusual event)				

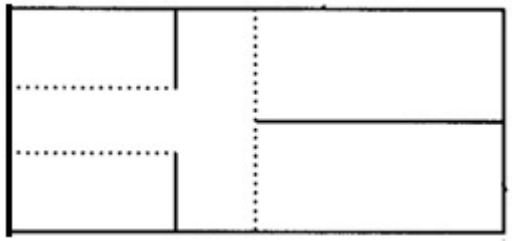


Helicopter Designs V through VIII

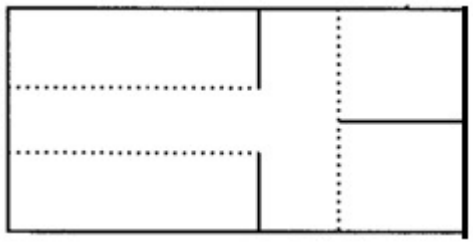
Run Order						Flight Time
		Short	*Small*			
		Rotor	*Frame*	**Clip**	*Sing*	
Model	Standard	-	-	+	+	
V	Order	See back (unusual event)				



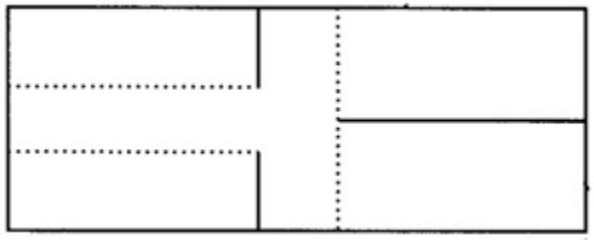
Run Order						Flight Time
		Long	*Small*		*No*	
		Rotor	*Frame*	**Clip**	*Sing*	
Model	Standard	+	-	+	-	
VI	Order		See back (unusual event)			



Run Order						Flight Time
		Short	*Big*		*No*	
		Rotor	*Frame*	**Clip**	*Sing*	
Model	Standard	-	+	+	-	
VII	Order	See back (unusual event)				



Run Order						Flight Time
		Long	*Big*			
		Rotor	*Frame*	**Clip**	*Sing*	
Model	Standard	+	+	+	+	
VIII	Order		See back (unusual event)			



One way to randomize (Accesses the Psychomotor Domain): Put each recipe card in a container

Run Order	Short Rotor	*Small Frame*	No Clip	*No Sing*	Flight Time
Model I / Standard Order	-	-	-	-	See back (unusual event)

Run Order	Long Rotor	*Small Frame*	No Clip	*Sing*	Flight Time
Model II / Standard Order	+	-	-	+	See back (unusual event)

Run Order	Short Rotor	*Big Frame*	No Clip	*Sing*	Flight Time
Model III / Standard Order	-	+	-	+	See back (unusual event)

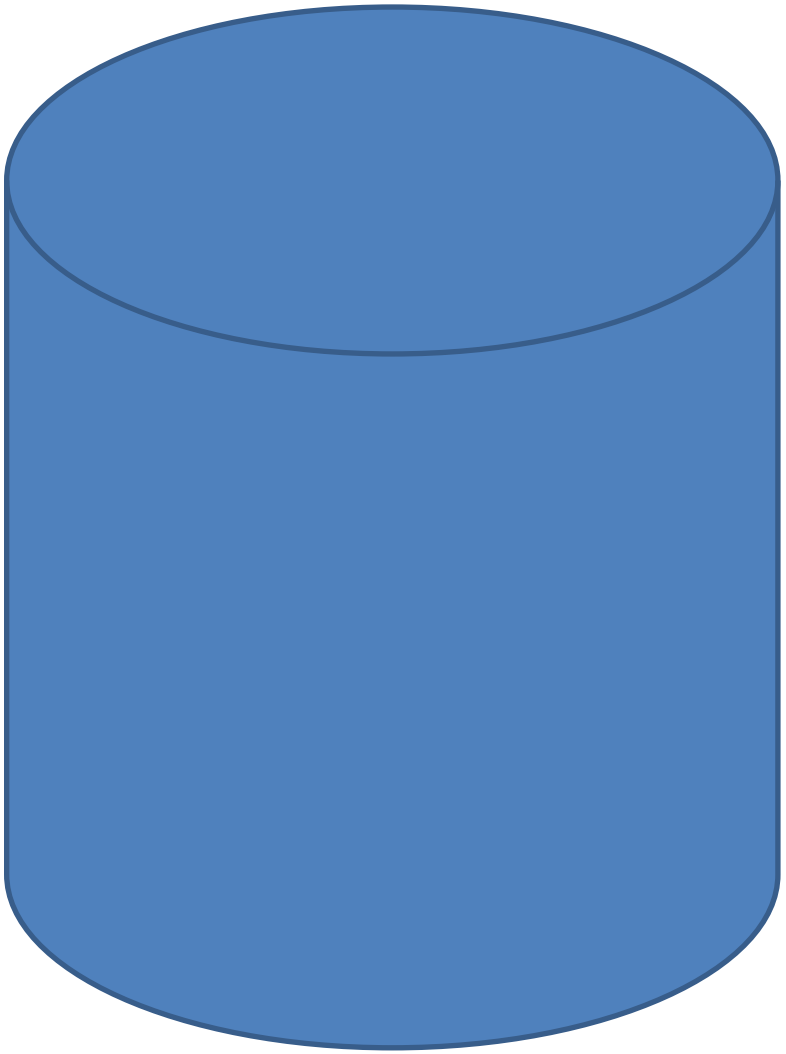
Run Order	Long Rotor	*Big Frame*	No Clip	*No Sing*	Flight Time
Model IV / Standard Order	+	+	-	-	See back (unusual event)

Run Order	Short Rotor	*Small Frame*	Clip	*Sing*	Flight Time
Model V / Standard Order	-	-	+	+	See back (unusual event)

Run Order	Long Rotor	*Small Frame*	Clip	*No Sing*	Flight Time
Model VI / Standard Order	+	-	+	-	See back (unusual event)

Run Order	Short Rotor	*Big Frame*	Clip	*No Sing*	Flight Time
Model VII / Standard Order	-	+	+	-	See back (unusual event)

Run Order	Long Rotor	*Big Frame*	Clip	*Sing*	Flight Time
Model VIII / Standard Order	+	+	+	+	See back (unusual event)

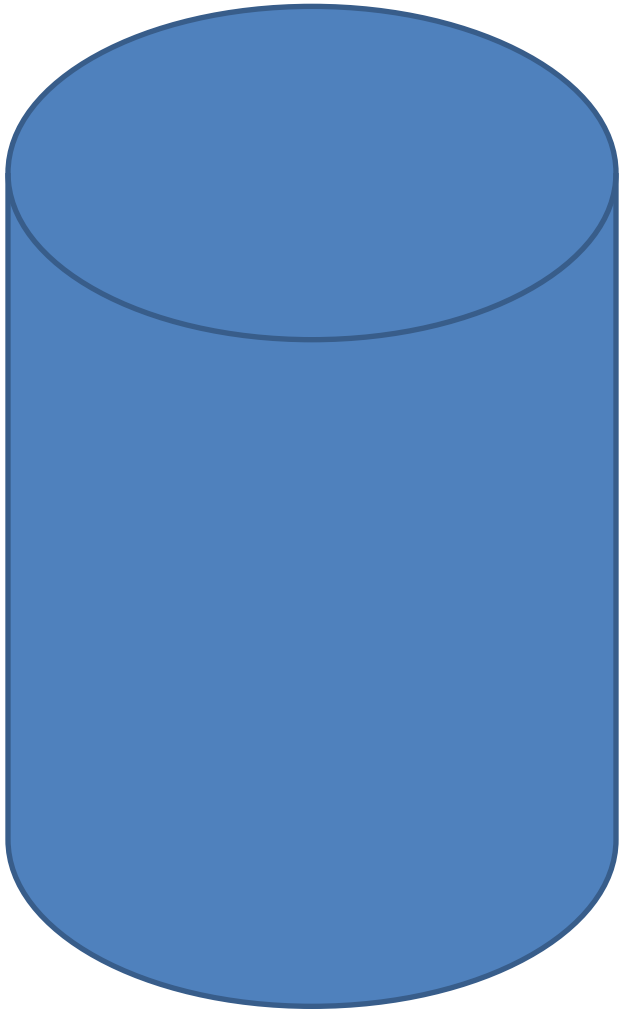


Draw a “Recipe Card”

Write the Order Drawn on The Card

Stir and Repeat Until All Recipe Cards Are Drawn

Run the Experiments In This Order

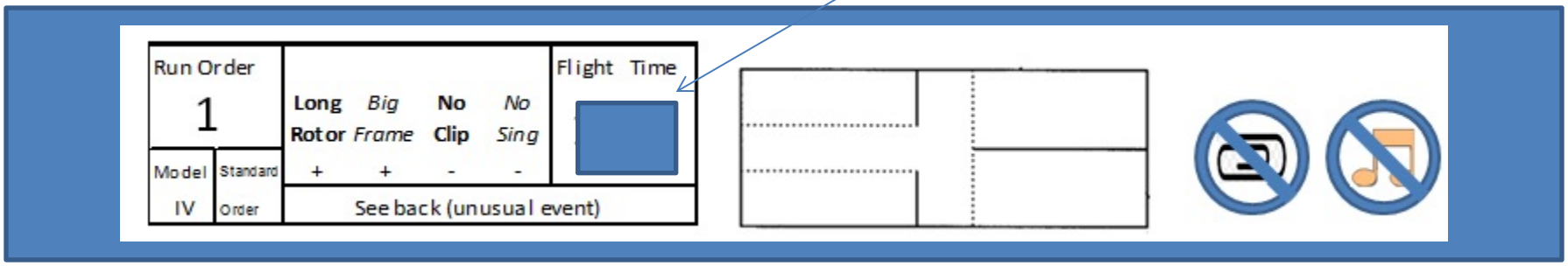


Run #1


1.85

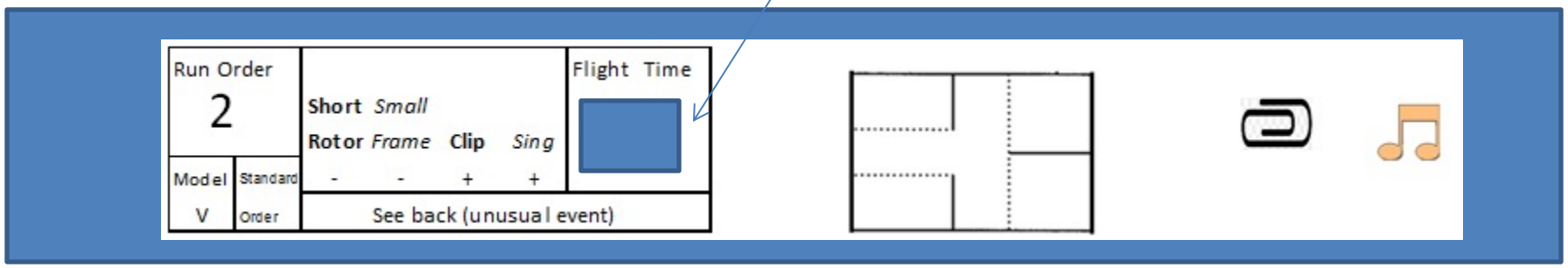

1.79


1.81

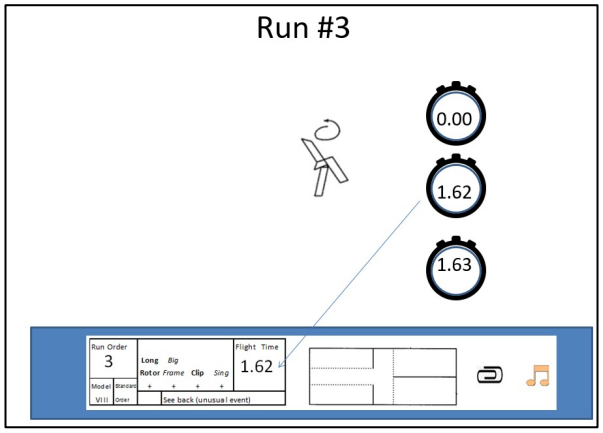

Run Order
1
Model
IV
Standard
Order
Long
Rotor
Big
Frame
No
Clip
No
Sing
+
+
-
-
Flight Time
See back (unusual event)

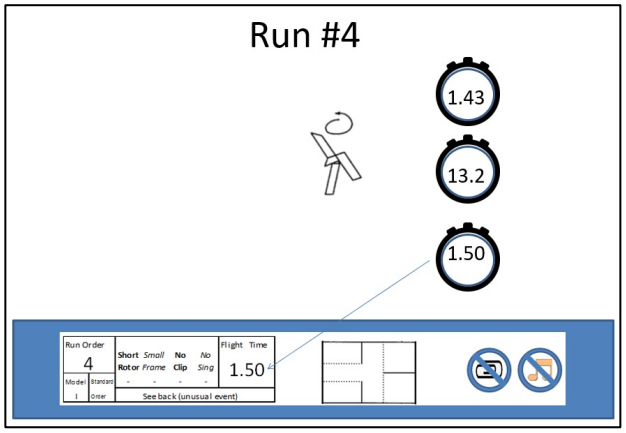
Run #2

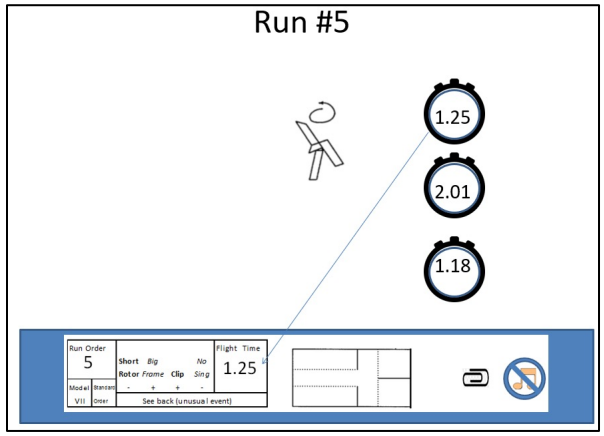

1.43
0.00
1.78

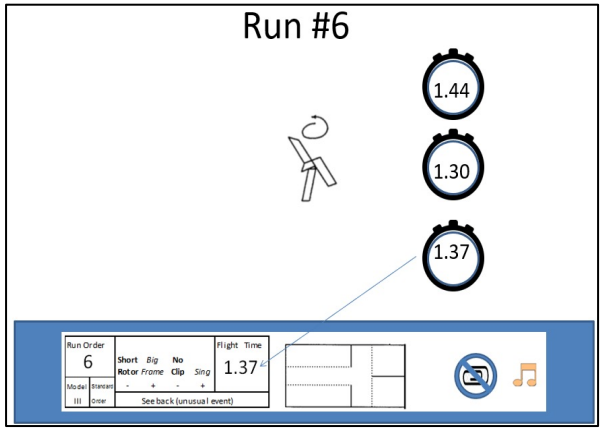

Run Order
2
Model
V
Standard
Order
Short Small
Rotor Frame Clip Sing
- - + +
See back (unusual event)
Flight Time

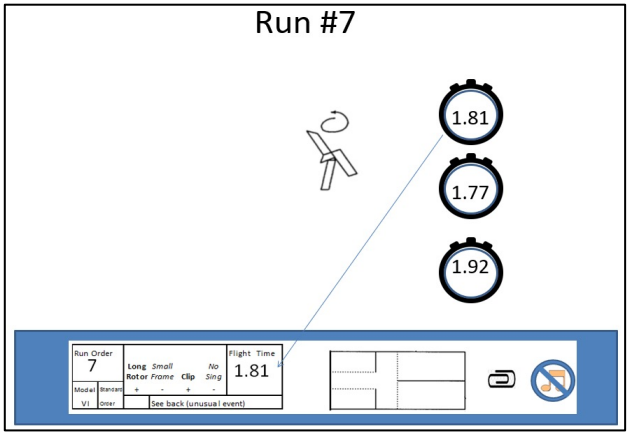
Repeat for Runs 3-8
Remember: Each Is a Different Helicopter Design

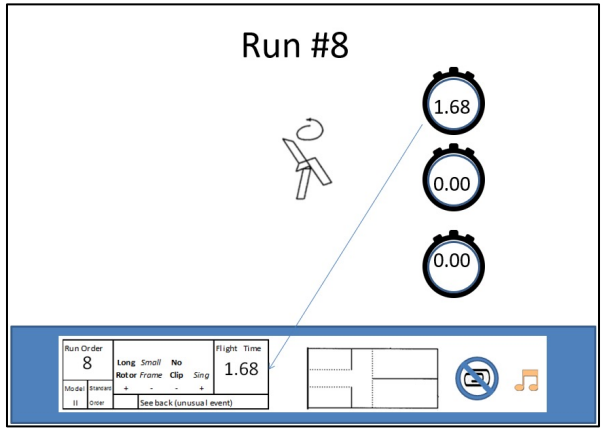




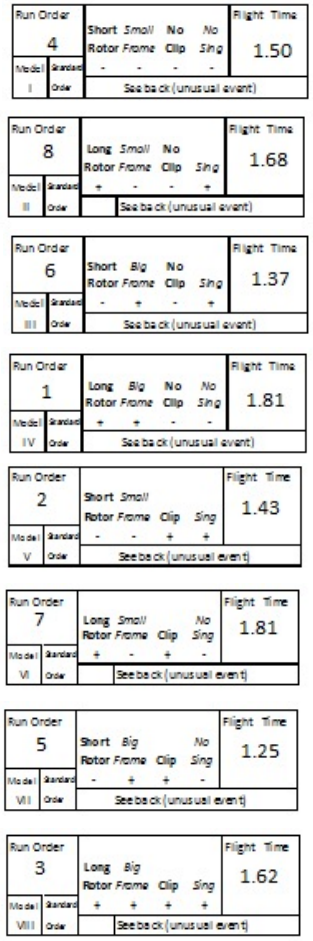








Put “Recipe Cards” In Standard Order
Calculate the Grand Average

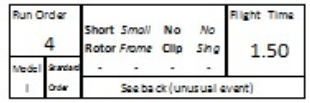


Run Order	Rotor	Frame	Clip	Sing	Flight Time	Model	Standard Order
4	Short	Small	No	No	1.50	I	See back (unusual event)
	-	-	-	-			
8	Long	Small	No	Sing	1.68	II	See back (unusual event)
	+	-	-	+			
6	Short	Big	No	Sing	1.37	III	See back (unusual event)
	-	+	-	+			
1	Long	Big	No	No	1.81	IV	See back (unusual event)
	+	+	-	-			
2	Short	Small			1.43	V	See back (unusual event)
	-	-	Clip	Sing			
			+	+			
7	Long	Small		No	1.81	VI	See back (unusual event)
	+	-	Clip	Sing			
			+	-			
5	Short	Big		No	1.25	VII	See back (unusual event)
	-	+	Clip	Sing			
			+	-			
3	Long	Big			1.62	VIII	See back (unusual event)
	+	+	Clip	Sing			
			+	+			

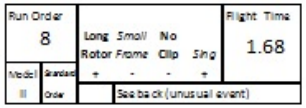
Total 12.47

Average
(Total/8) 1.155875

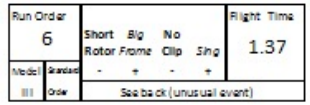
Next, Group By Rotor Length



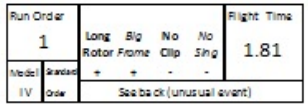
Run Order	Rotor	Frame	Clip	Sing	Flight Time
4	Short	Small	No	No	1.50
Model Standard Order	-	-	-	-	
I	See back (unusual event)				



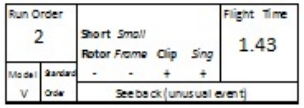
Run Order	Rotor	Frame	Clip	Sing	Flight Time
8	Long	Small	No		1.68
Model Standard Order	+	-	-	+	
II	See back (unusual event)				



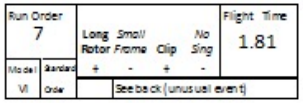
Run Order	Rotor	Frame	Clip	Sing	Flight Time
6	Short	Big	No		1.37
Model Standard Order	-	+	-	+	
III	See back (unusual event)				



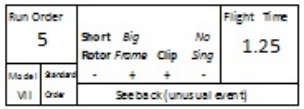
Run Order	Rotor	Frame	Clip	Sing	Flight Time
1	Long	Big	No	No	1.81
Model Standard Order	+	+	-	-	
IV	See back (unusual event)				



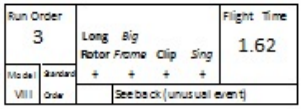
Run Order	Rotor	Frame	Clip	Sing	Flight Time
2	Short	Small			1.43
Model Standard Order	-	-	+	+	
V	See back (unusual event)				



Run Order	Rotor	Frame	Clip	Sing	Flight Time
7	Long	Small		No	1.81
Model Standard Order	+	-	+	-	
VI	See back (unusual event)				



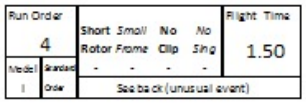
Run Order	Rotor	Frame	Clip	Sing	Flight Time
5	Short	Big		No	1.25
Model Standard Order	-	+	+	-	
VII	See back (unusual event)				

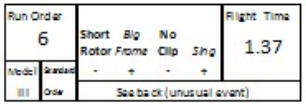


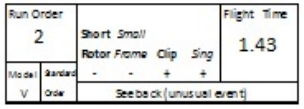
Run Order	Rotor	Frame	Clip	Sing	Flight Time
3	Long	Big			1.62
Model Standard Order	+	+	+	+	
VIII	See back (unusual event)				

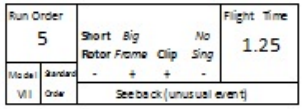
CALCULATE MAIN EFFECT OF DIFFERENT ROTOR LENGTHS

Short Rotor

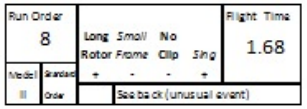


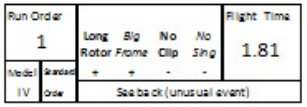


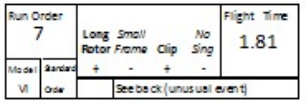


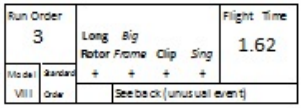


Long Rotor



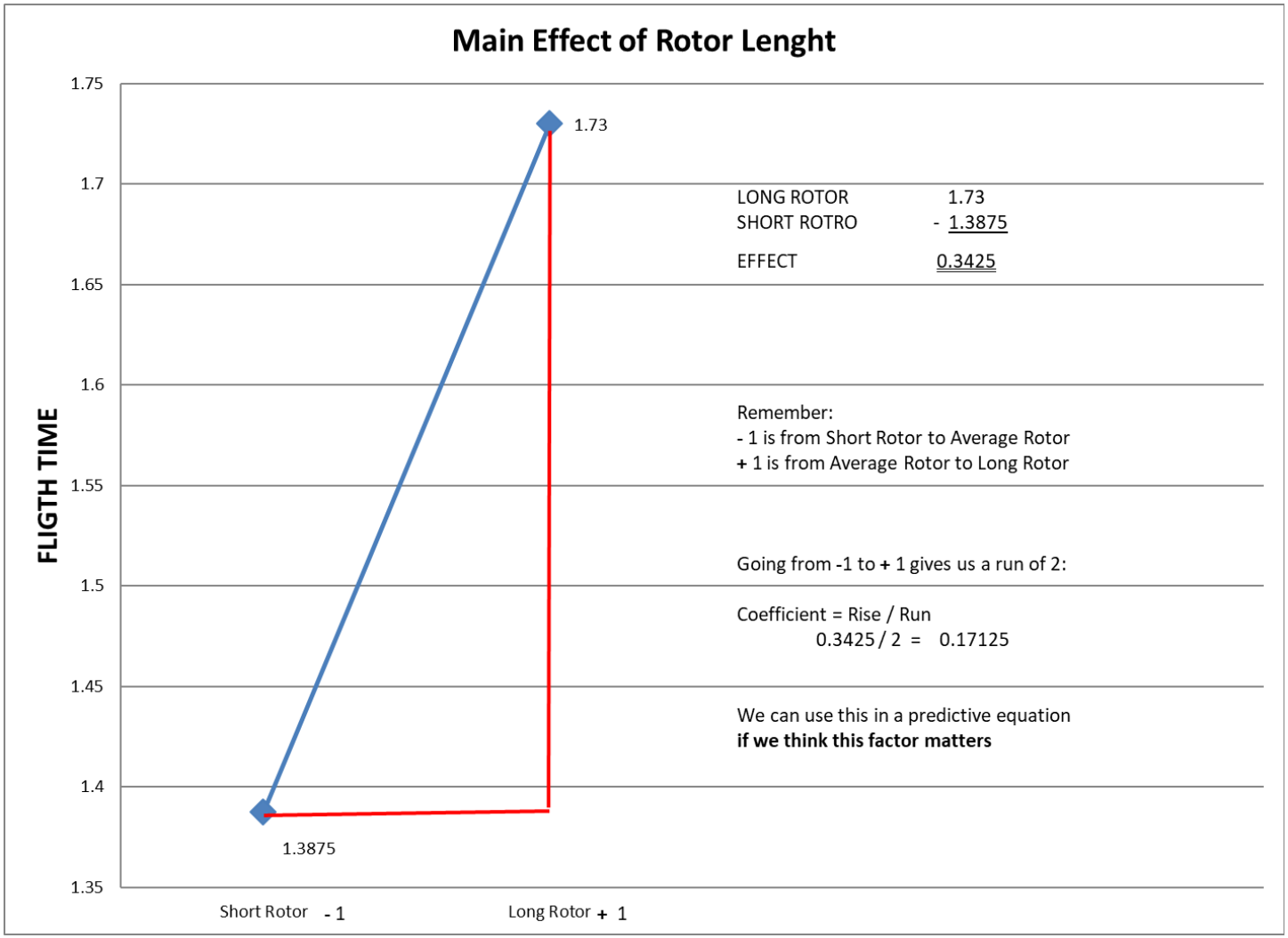






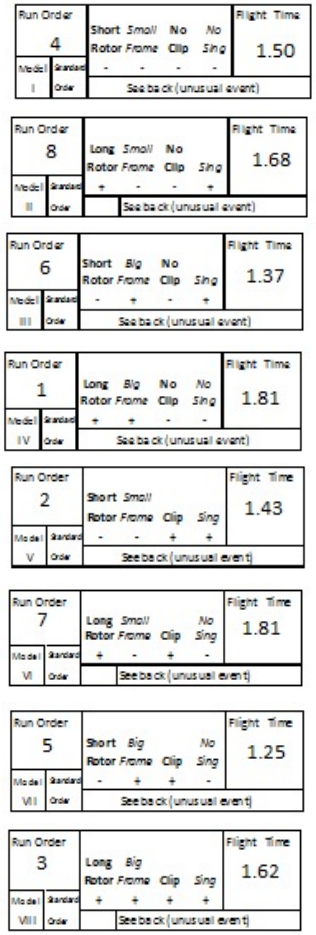
Total	5.55		6.92
Avg. (Total / 4)	1.3875		1.730
Effect (Long – Short)		0.3425	
Coefficient (Effect / 2)		0.17125	

Why We Divide The Effect by 2 (*in this example)* To Obtain Equation Coefficients



CALCULATE MAIN EFFECT OF DIFFERENT FRAME SIZES

Small Frame



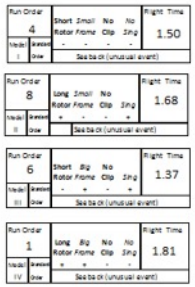
Big Frame

Total	6.42		6.05
Avg. (Total / 4)	1.605		1.513
Effect (Long Avg. – Short Avg.)		-0.0925	
Coefficient (Effect / 2)		-0.04625	

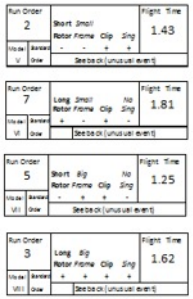
Calculate Other Two Main Effects

CALCULATE THE MAIN EFFECT OF A PAPER CLIP

No Clip



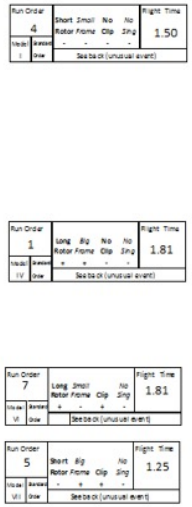
Clip



Total	6.36		6.11
Avg, Total / 4)	1.590		1.528
Effect (Long Avg. – Short Avg.)		-0.0625	
Coefficient (Effect / 2)		-0.03125	

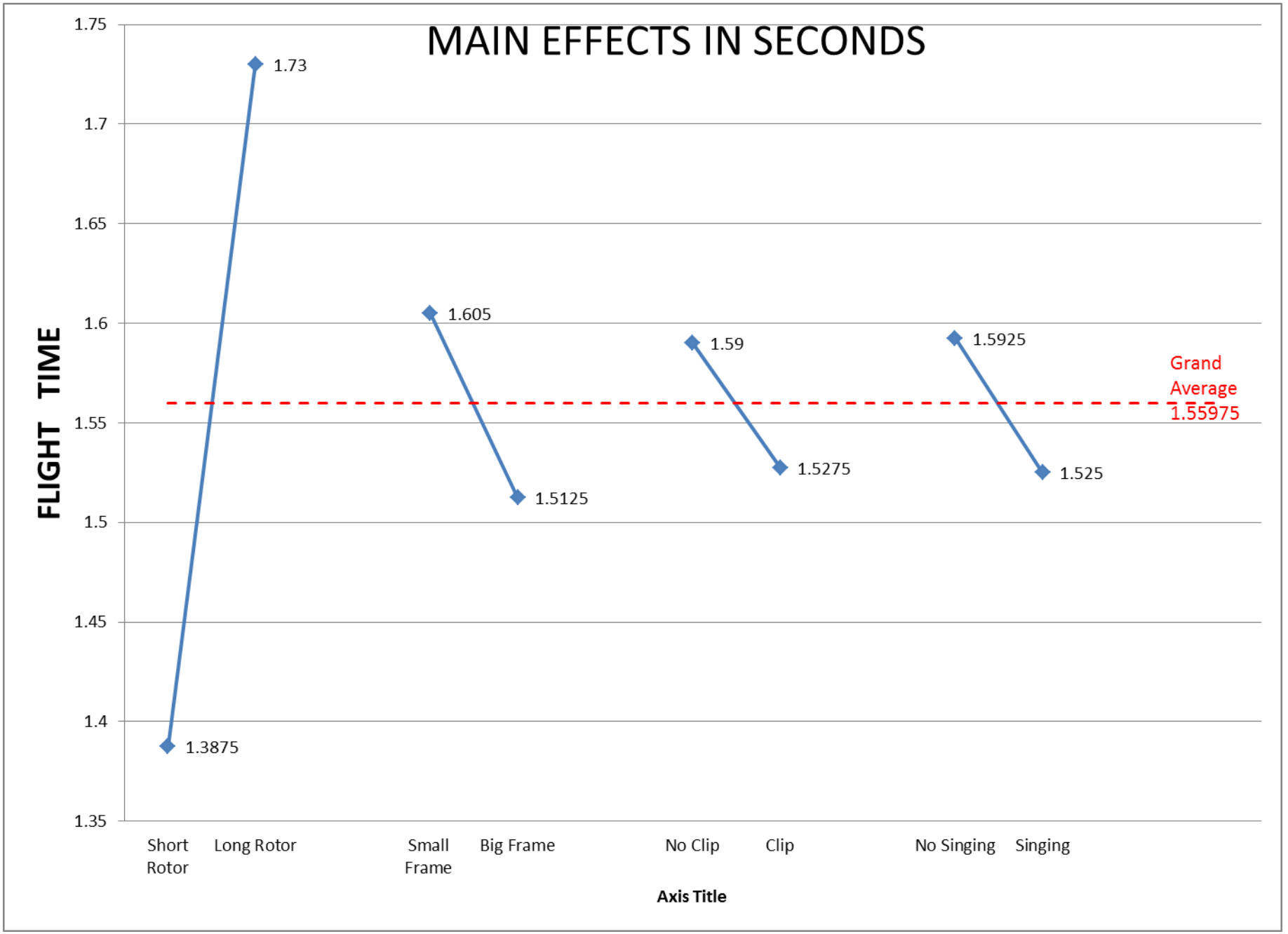
CALCULATE THE MAIN EFFECT OF SINGING

No Singing



Singing

Total	6.437		6.1
Avg. (Total / 4)	1.5925		1.525
Effect (Long Avg. – Short Avg.)		-0.0675	
Coefficient (Effect / 2)		-0.03375	


MAIN EFFECTS IN SECONDS
FLIGHT TIME
1.75
1.7
1.65
1.6
1.55
1.5
1.45
1.4
1.35
1.73
1.3875
1.605
1.5125
1.59
1.5275
1.5925
1.525
Grand Average 1.55975
Short Rotor
Long Rotor
Small Frame
Big Frame
No Clip
Clip
No Singing
Singing
Axis Title

The Helicopter Experiment

- "The Helicopter Experiment" is used extensively in 6-Sigma/Quality/Statistics training in industry
- More Sophisticated Versions:
  - Key Source:
    - *George Box, FRS:*
      - Teaching Engineers Experimental Design With A Paper Helicopter, *Quality Engineering,* 1992, Vol. 4, No. 3, pp. 453-459
      - Also Report No. 76 of the Center for Quality and Productivity Improvement of the University of Wisconsin
  - Slightly more complex Paper Helicopter Experiment:
    - *Matthew Barsalou:*
      - Teaching DoE with Paper Helicopters and Minitab http://www.minitab.com/en-us/Published-Articles/Teaching-DoE-with-Paper-Helicopters-and-Minitab/
  - More Advance Paper Helicopter Experiment
    - *Erik Barry Erhardt*
      - Designing a Better Paper Helicopter Using Response Surface Methodology. *Stats*, Issue 48, pp. 14-19, also see p. 2.
  - Statistical Software:
    - DesignExpert https://www.statease.com/
    - JMP https://www.jmp.com/en_us/applications/design-of-experiments.html
    - Minitab http://www.minitab.com/en-us/

More Information

- Box, G. E. P. (1992), "Teaching Engineers Experimental Design with a Paper Helicopter," *Quality Engineering*, 4, 453–459. Found at Teaching Engineers Experimental Design with a Paper Helicopter (williamghunter.net) also see Learning Design of Experiments with Paper Helicopters » Curious Cat Science and Engineering Blog (curiouscatblog.net)
- and for a more advanced optimization approach see Erik Barry Erhardt "Designing a Better Paper Helicopter Using Response Surface Methodology," *Stats* Issue 48, pp. 14-21.
- 2004-08Stat-Teaser.qxd (statease.com)
- Creativity defeats sensibility for paper helicopter fly-off « Stats Made Easy
  - Video of paper-helicopter fly-offs at South Dakota School of Mines & Technology « Stats Made Easy
- Learning Design of Experiments with Paper Helicopters and Minitab

Discussion

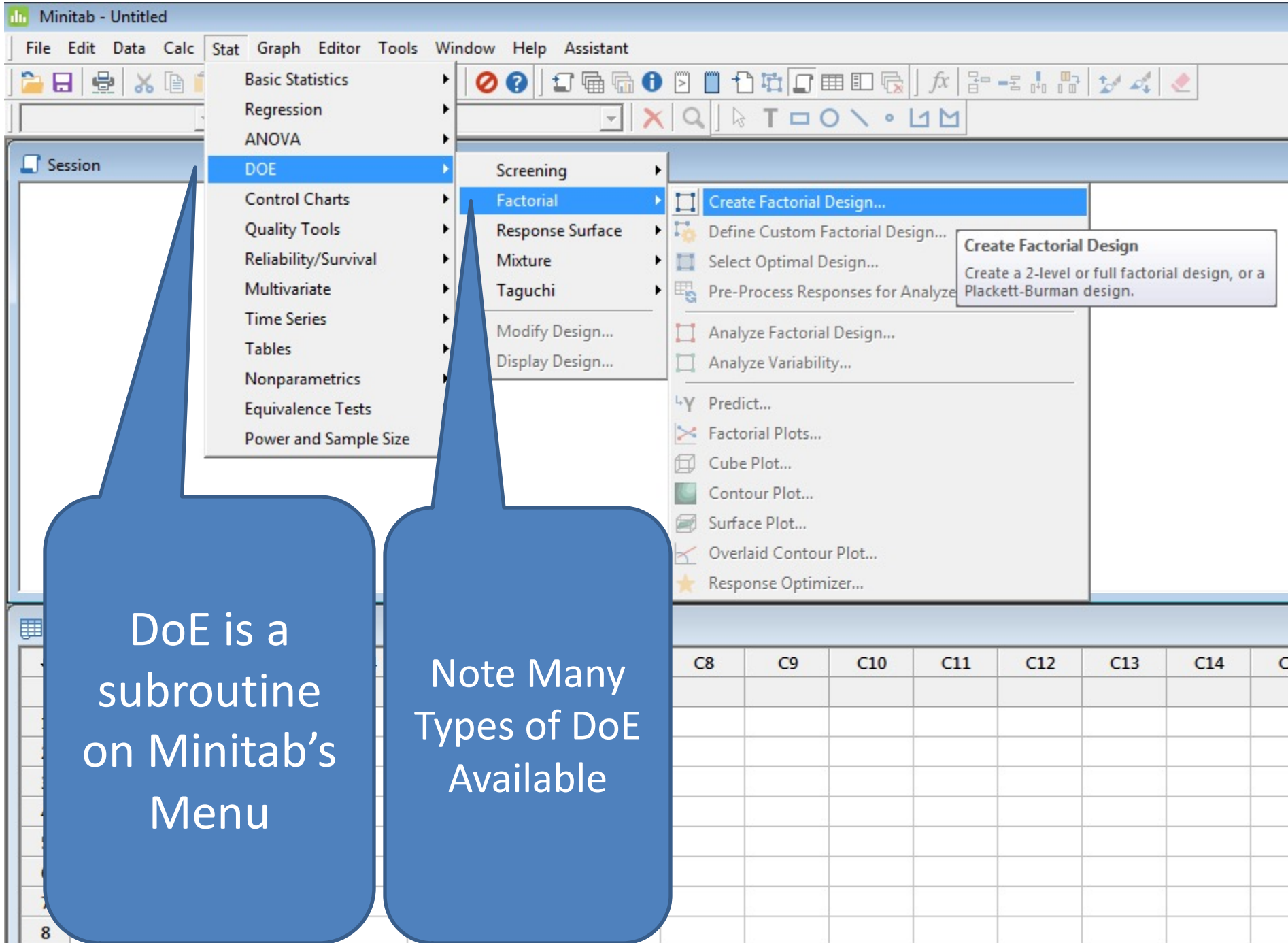
- **Should Introductory Statistics Students See a Demonstration—or Preferably Participate in the Planning, Execution, and Analysis—of a Simplified Multifactor Statistically Design of Experiments (DoE)?**
- **What About Students Who Do Not Take Stat 101?**
- **Is the Recipe Card Approach A Good Example of How To Introduce DoE?**
  - **How Can It Be Improved?**
- **Are You Interested In Continuing the Conversation?**
  - fluharty.earlydoe@gmail.com

Appendix I

SOFTWARE IMPLEMENTATION

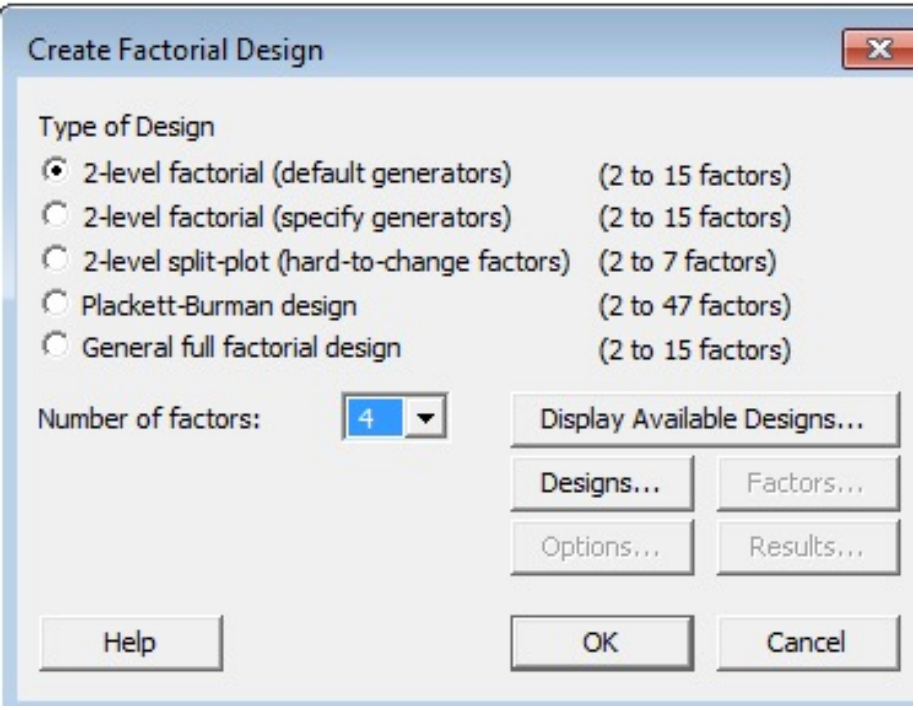
Using Software for DoE

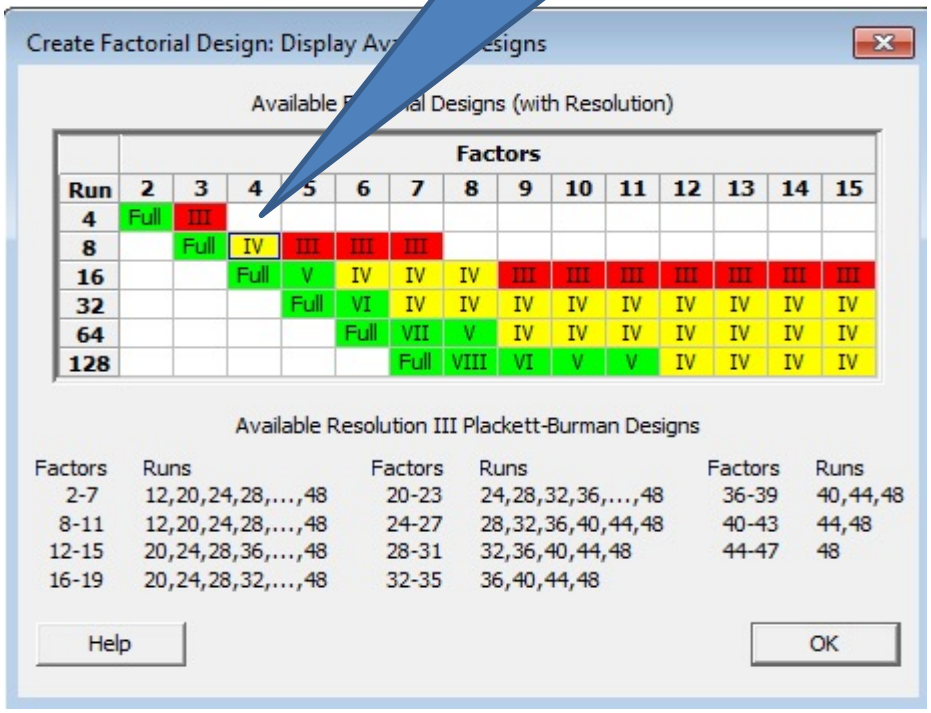
- Software: Among the Statistical Software Programs With Extensive DoE Implementations Are the Following:
  - Minitab (Used In This Example, USCOTS 2021 Sponson)
  - JMP(USCOTS 2021 Sponson)
  - SAS (USCOTS 2021 Sponson)
  - StatEase Design Expert (Specialized DoE Software)
- This Example Is an Intermediate Model to Show Interaction
  - Most Experimenters Would Develop A Reduced Model Which Would Probably Not Include Interaction and Several Other Terms


Minitab - Untitled
File Edit Data Calc Stat Graph Editor Tools Window Help Assistant
Basic Statistics
Regression
ANOVA
DOE
Control Charts
Quality Tools
Reliability/Survival
Multivariate
Time Series
Tables
Nonparametrics
Equivalence Tests
Power and Sample Size
Screening
Factorial
Response Surface
Mixture
Taguchi
Modify Design...
Display Design...
Create Factorial Design...
Define Custom Factorial Design...
Select Optimal Design...
Pre-Process Responses for Analyze
Analyze Factorial Design...
Analyze Variability...
Predict...
Factorial Plots...
Cube Plot...
Contour Plot...
Surface Plot...
Overlaid Contour Plot...
Response Optimizer...
Create Factorial Design
Create a 2-level or full factorial design, or a Plackett-Burman design.
Session
C8 C9 C10 C11 C12 C13 C14
DoE is a subroutine on Minitab's Menu
Note Many Types of DoE Available

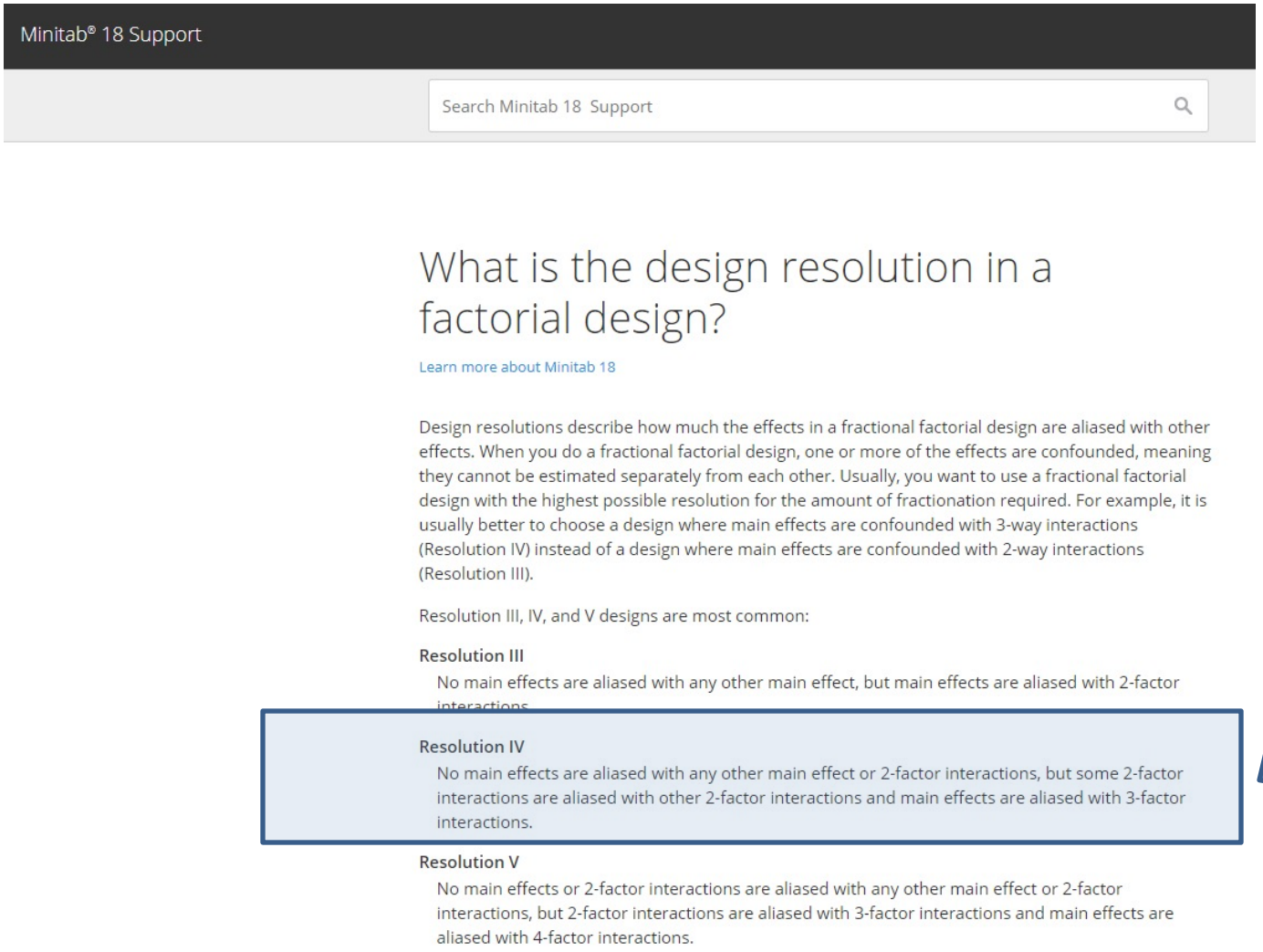
Menu Guides Experimenter Through Design Generation

This Table Tells Us That a 4 Factor 8 Run Design Is Resolution IV



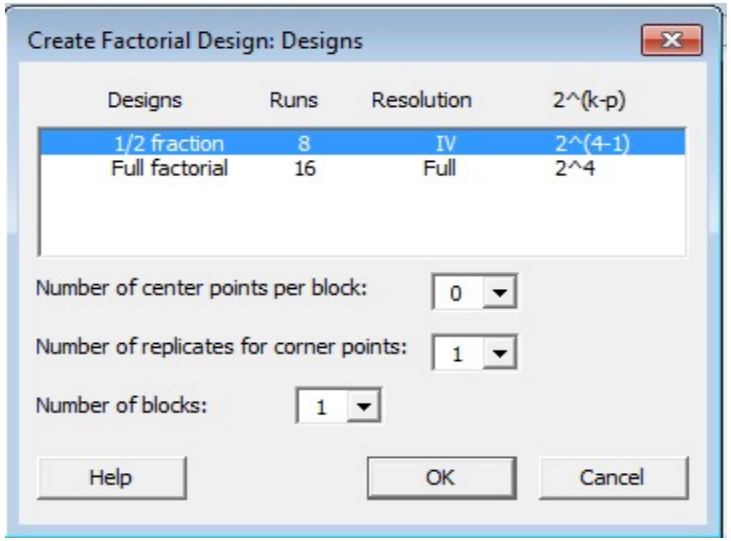


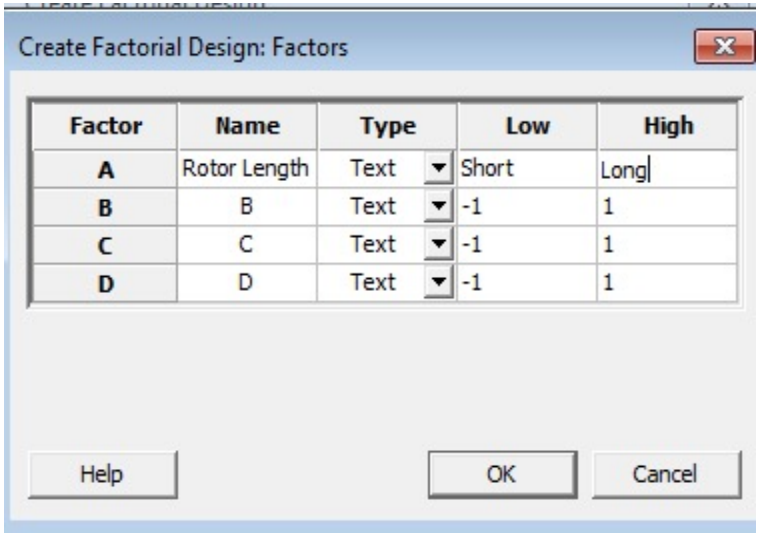
What We Give Up When We Use Fractional Factorials

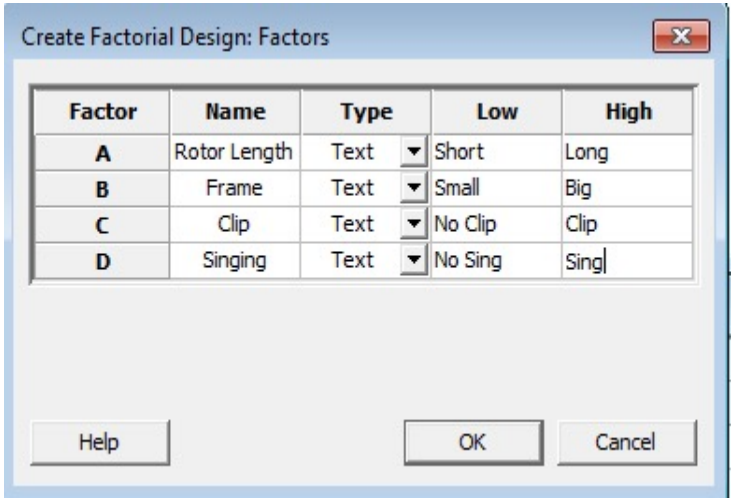


Resolution IV
No main effects are aliased with any other main effect or 2-factor interactions, but some 2-factor interactions are aliased with other 2-factor interactions and main effects are aliased with 3-factor interactions.

Specifying the Design and Naming the Factor Levels







Factorial Regression: Flight Time versus Rotor Length, Frame, Clip, Singing

Coded Coefficients

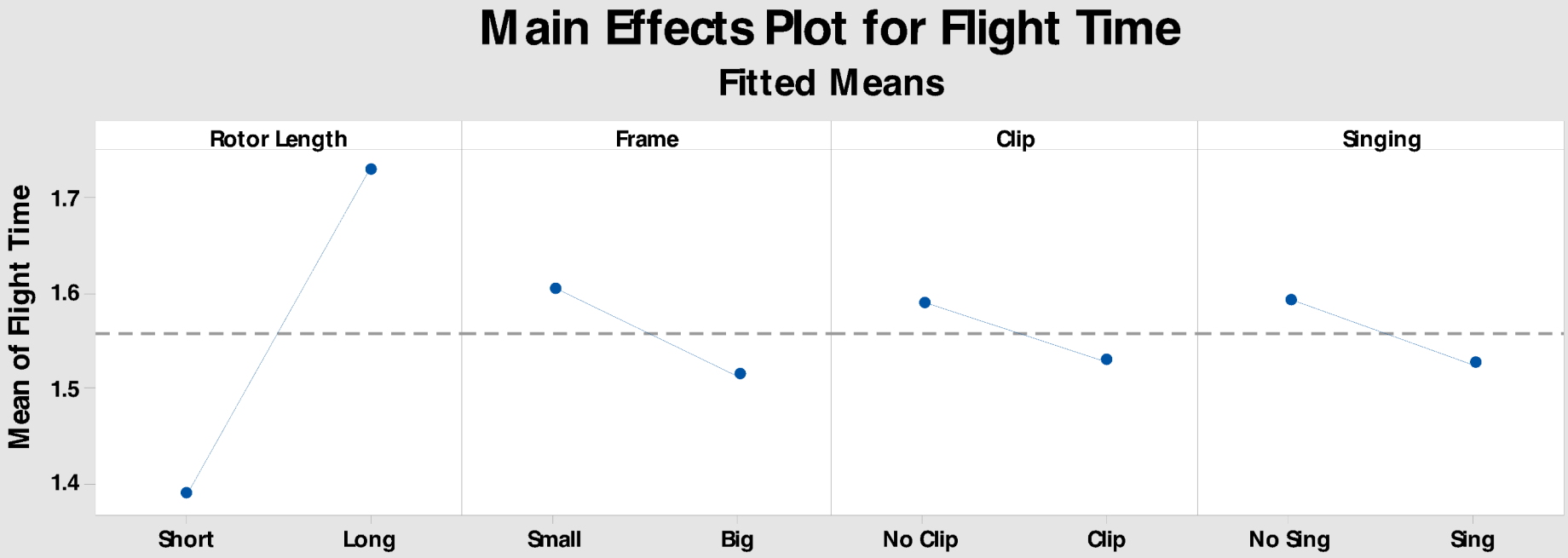
Term	Effect	Coef	SE Coef
Constant		1.5588	0.0249
Rotor Length	0.3425	0.1713	0.0249
Frame	-0.0925	-0.0462	0.0249
Clip	-0.0625	-0.0313	0.0249
Singing	-0.0675	-0.0338	0.0249
Frame*Clip	-0.0925	-0.0463	0.0249

The Coef(ficients) can be used to develop predictive equations, including interactions. Experimenter would decide which terms to include.

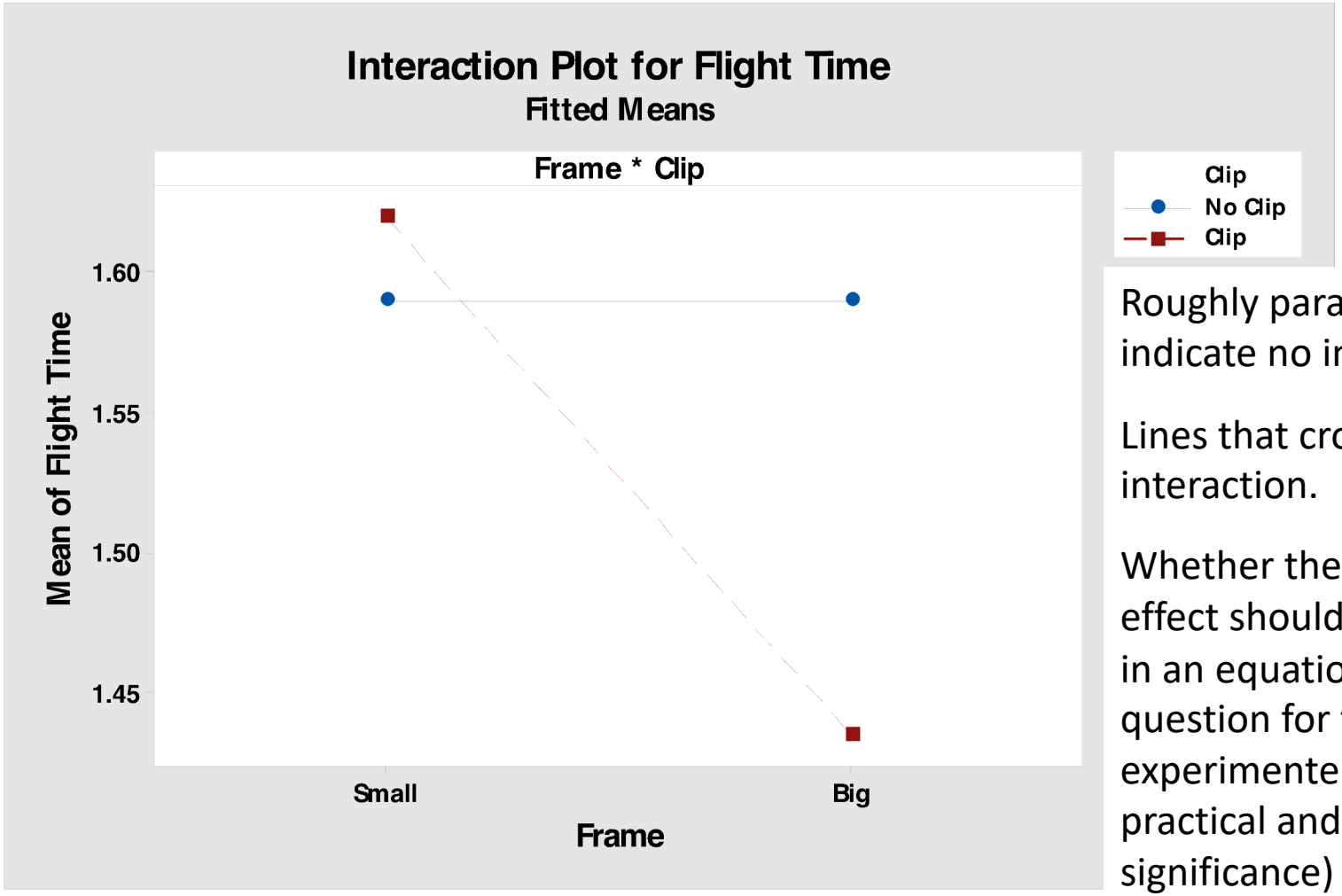
NOTE: A reduced model would probably not include any Terms except the Constant and Rotor Length

A likely "reduced form" equation
Flight Time = Average + or – Times Coefficient for Rotor Length
Flight Time = 1.5588 + [(+ if Long, - if Short) x 0.1713
For Long Rotor: Flight Tim = 1.5588 + 0.1713
For Short Rotor: Flight Time = 1.5588 - 0.1713

Plot of Results



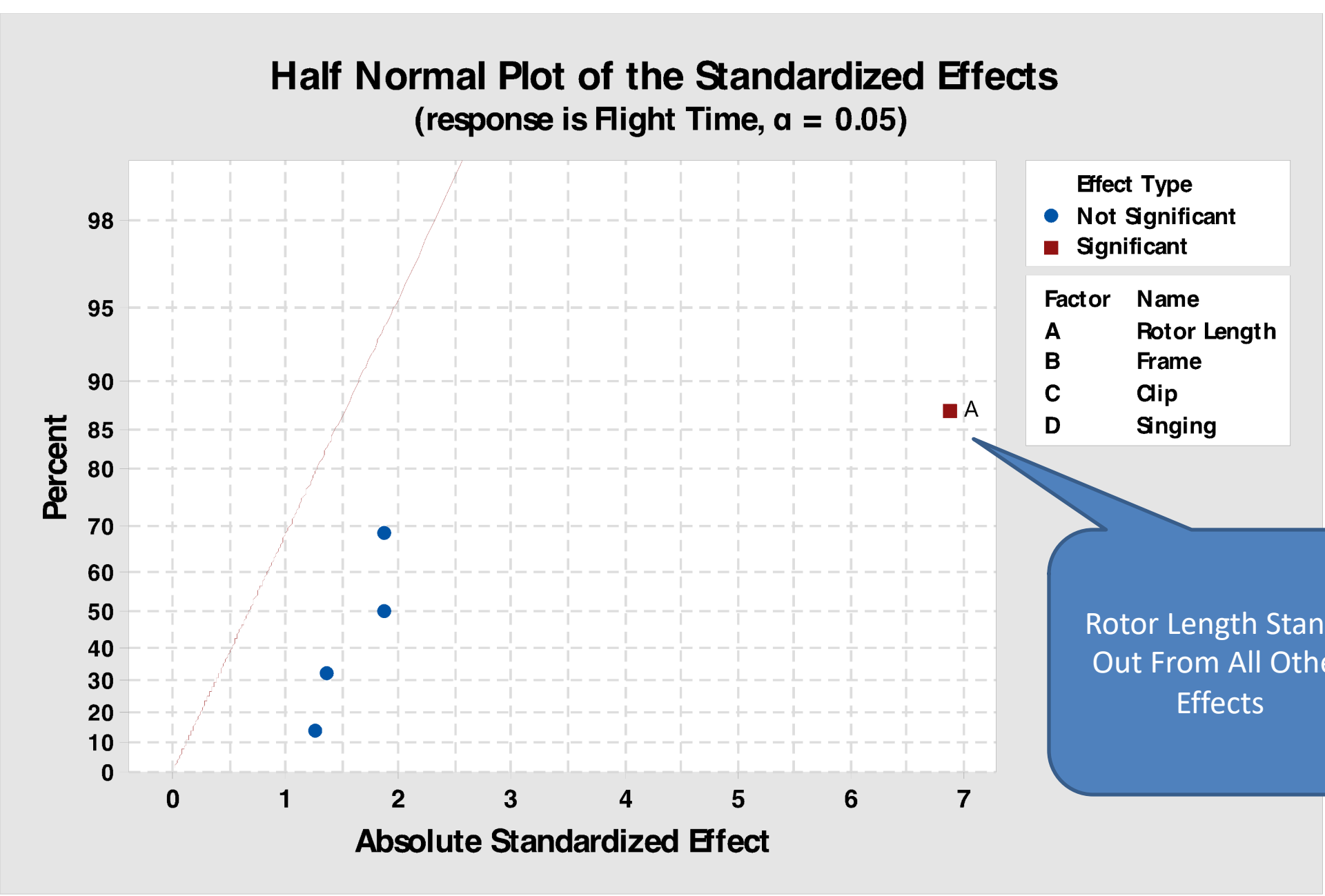
This Interaction Plot Splits the Main Effect Plot for Frame Size Into Two Lines: One Where There Is a Clip, The Other Where There Is Not a Clip

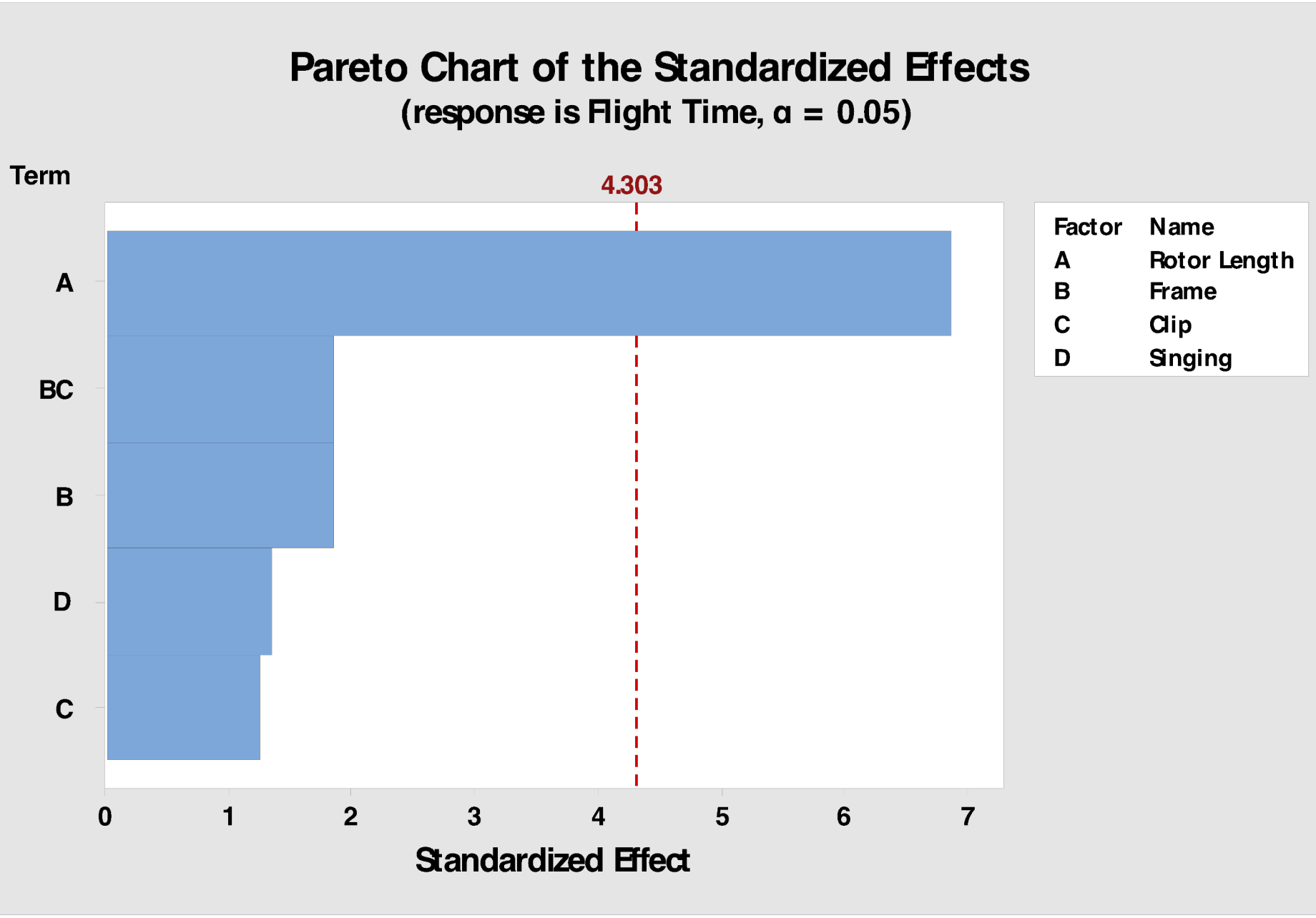


Roughly parallel lines indicate no interaction.

Lines that cross indicate an interaction.

Whether the interaction effect should be included in an equation is a question for the experimenter (questions of practical and statistical significance)


Half Normal Plot of the Standardized Effects
(response is Flight Time, α = 0.05)
Percent
98
95
90
85
80
70
60
50
40
30
20
10
0
0
1
2
3
4
5
6
7
Absolute Standardized Effect
A
Effect Type
Not Significant
Significant
Factor Name
A Rotor Length
B Frame
C Clip
D Singing
Rotor Length Stan
Out From All Othe
Effects


Pareto Chart of the Standardized Effects
(response is Flight Time, α = 0.05)
Term
4.303
A
BC
B
D
C
0
1
2
3
4
5
6
7
Standardized Effect
Factor Name
A Rotor Length
B Frame
C Clip
D Singing

Appendix II

SHORT OUTLINE OF SELECTED STEPS IN A DOE

Short Outline of Selected Steps in a DoE (1 of 2)

1. Define the problem or opportunity
2. Identify the response (what are you interested in)
3. Specify the objective(s) for the response (minimize, hit a value, maximize, etc.)
   - Are there tradeoffs between objectives, e.g., flight time and land in a specific location?
4. Identify potential causes
   - Brainstorm
   - Research
   - Literature Search
   - Textbooks
5. Classify potential causes as control (can be changed relatively easily in practice) and noise (can not be changed in practice)
   - This distinction is very important in 'Robust Design"
6. Develop experimental strategy
   - Screening/Optimization/Confirmation
   - Includes budget (time, materials, money)
   - Perhaps 25% devoted to initial screening experiment
7. Choose the experimental factors and select factor levels (may be iterative with the "Generate the DoE" step
8. Identify restrictions and constraints on factors and randomization

Short Outline of Selected Steps in a DoE (2 of 2)

9. Ensure all experimental runs are safe to perform
10. Obtain IRB (Institutional Review Board) approval if necessary
11. Choose/Generate the DoE Plan with the response and chosen factor levels
    - The 4 factor 8 run example used in the USCOTS Beyond Session is one of many thousands of possible DoEs
    - Some of these DoEs have been used for years and can be found in standard texts—others can be generated by DoE software for specific purposes
12. Prepare to conduct the DoE
    - Time and place to run the experiments
    - Participant roles clear
13. Ensure the measurement system is capable
14. Randomize the runs
15. Run the DoE
    - Record information on unforeseen events that might have impacted the experiment
16. Collect the data
17. Analyze the data
18. Draw conclusions and decide on next steps

Subset of Slides 1-45

THE FOLLOWING SLIDES ARE PRESENTED AT THE USCOTS 2021 TUESDAY “BEYOND” SESSION TO START THE DISCUSSION

Should Introductory Statistics Classes Include Multifactor Statistical Design of Experiments (DoE)?

USCOTS 2021

Tuesday, June 29th 3:00-3:45pm ET

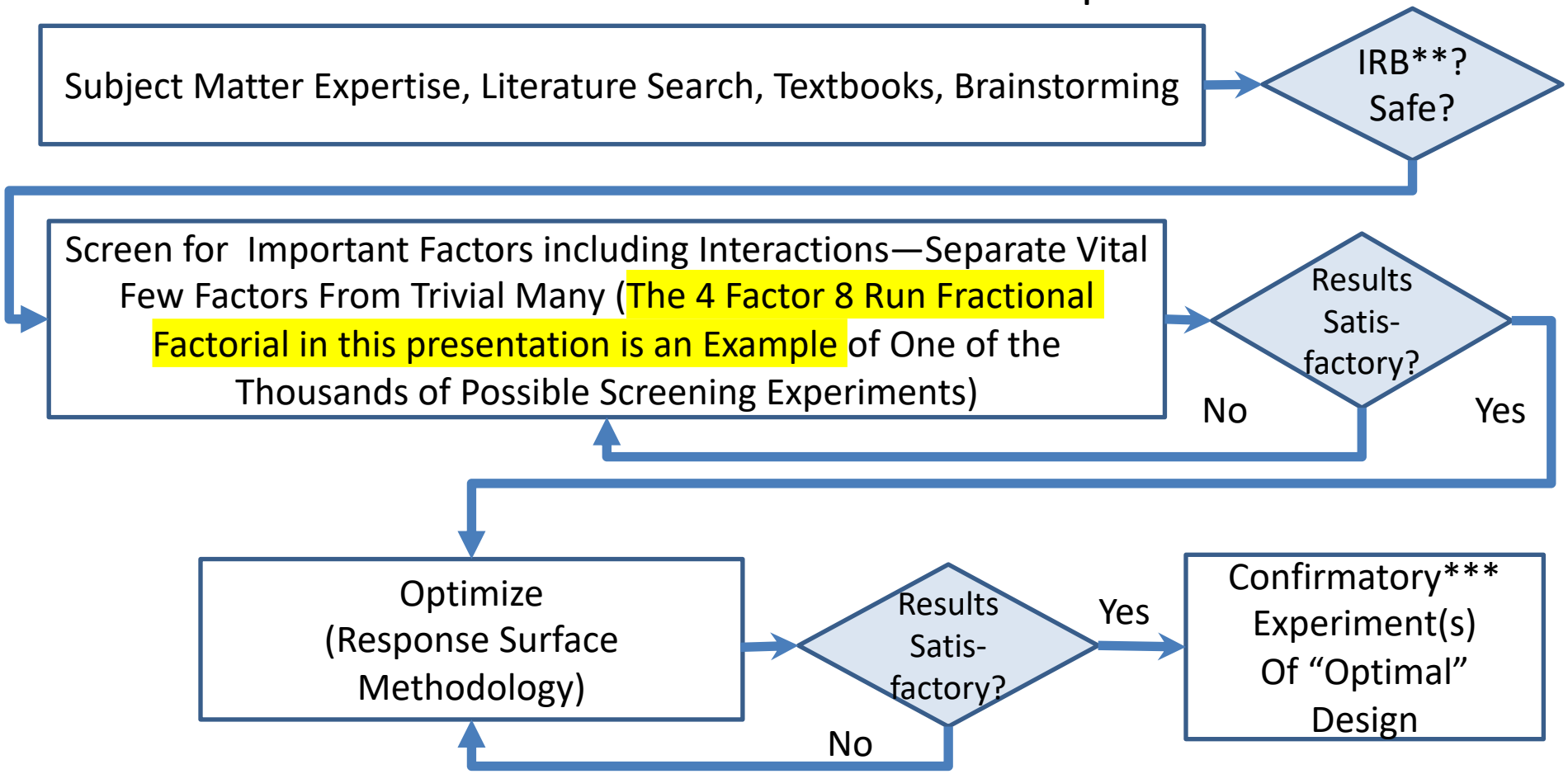
David Fluharty

Adjunct Professor of Statistics and Economics

Ivy Tech Community College—Columbus IN

fluharty.earlydoe@gmail.com

DoE Is an Art and Science that Helps Us Investigate the Effects of Multiple Factors (Including Interactions*) by **SIMULTANEOUSLY** Varying These Factors According to a Mathematically/Statistically **FIXED** Set of Recipes. Results Include Predictive Equations and Possibly Optimization. This Contrasts With One-Factor-At-A-Time Experimentation.

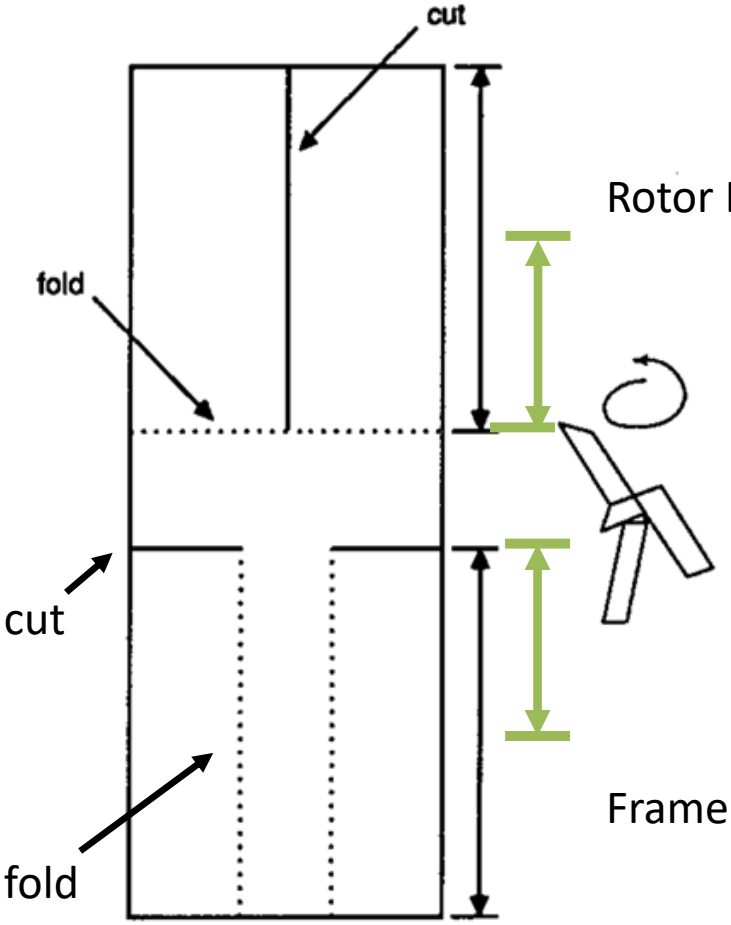


* Estimates of Interactions Depends on "Resolution" of Design

** Institutional Review Board

***Ensure "Optimal Result" Works as Expected (very important in industry}

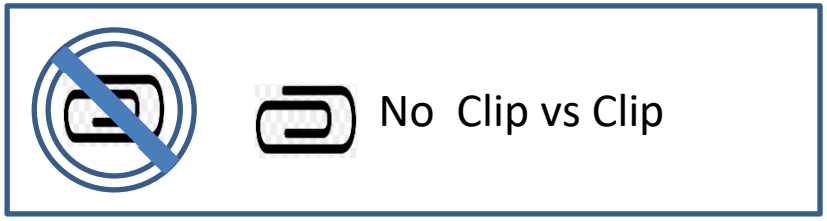
This Example Is Dropping a Paper Helicopter from A Ladder
We Investigate 4 Factors That ***Might*** Impact Flight Time:
Rotor Length (Long vs. Short), Frame Size (Big vs. Small), Paper Clip (No Clip vs. Clip), Singing (No Singing vs. Singing)



Rotor Length: Short and Long

No Singing vs Singing

Frame Size: Small and Big



No Clip vs Clip

Before Running the Experiments "Guesstimate" the Effects (Direction and Deviation from Average) and How Much of A Difference Matters

4 Factors 8 Runs (Each Different)
(Why This Works—Matrix Algebra)

Model Name	Rotor Length	Frame Size	Clip	Singing	"Standard Order"
I	Short	Small	No Clip	No Sing	**-1 -1 -1 -1**
II	Long	Small	No Clip	Sing	**+1 -1 -1 +1**
III	Short	Big	No Clip	Sing	**-1 +1 -1 +1**
IV	Long	Big	No Clip	No Sing	**+1 +1 -1 -1**
V	Short	Small	Clip	Sing	**-1 -1 +1 +1**
VI	Long	Small	Clip	No Sing	**+1 -1 +1 -1**
VII	Short	Big	Clip	No Sing	**-1 +1 +1 -1**
VIII	Long	Big	Clip	Sing	**+1 +1 +1 +1**

When Multiply the Rows, the Sum (Dot Product) is Zero➔ These are Orthogonal Vectors

The "Singing" Column is the product of the other three columns. For example, for Model VIII:

+1 x +1 x +1 = +1

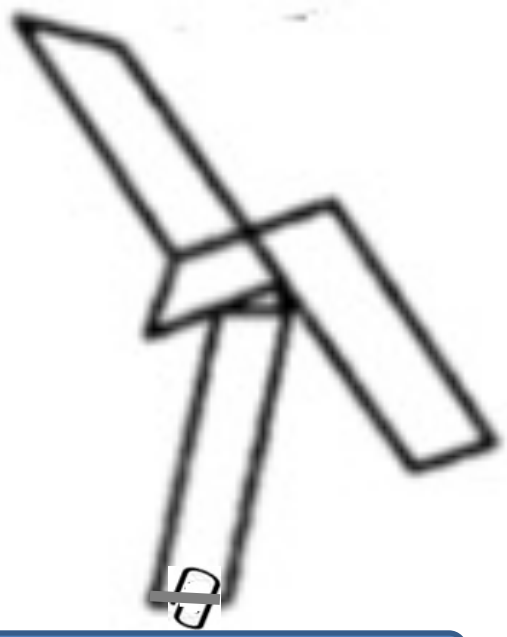
There is One Recipe Card Corresponding to Each of the Helicopter Designs: Example of Design VIII

This section contains the "recipe" for the helicopter design

The flight time of this helicopter is recorded here

The random run order is recorded here (can randomize by pulling out of a hat)

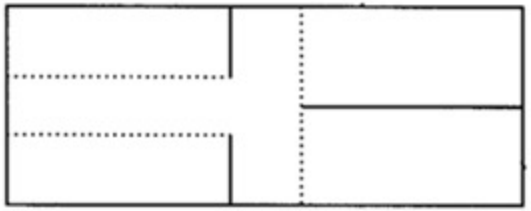
Run Order			Flight Time
3	Long Rotor +	Big Frame + Clip + Sing +	1.62
Model VIII	Standard Order	See back (unusual event)	



The model number (design name called "standard order" by statisticians

Good experimental practice is to record any unusual event during he experimental run for future analysis

Run Order			Flight Time
	Long Rotor +	Big Frame + Clip + Sing +	
Model VIII	Standard Order	See back (unusual event)	



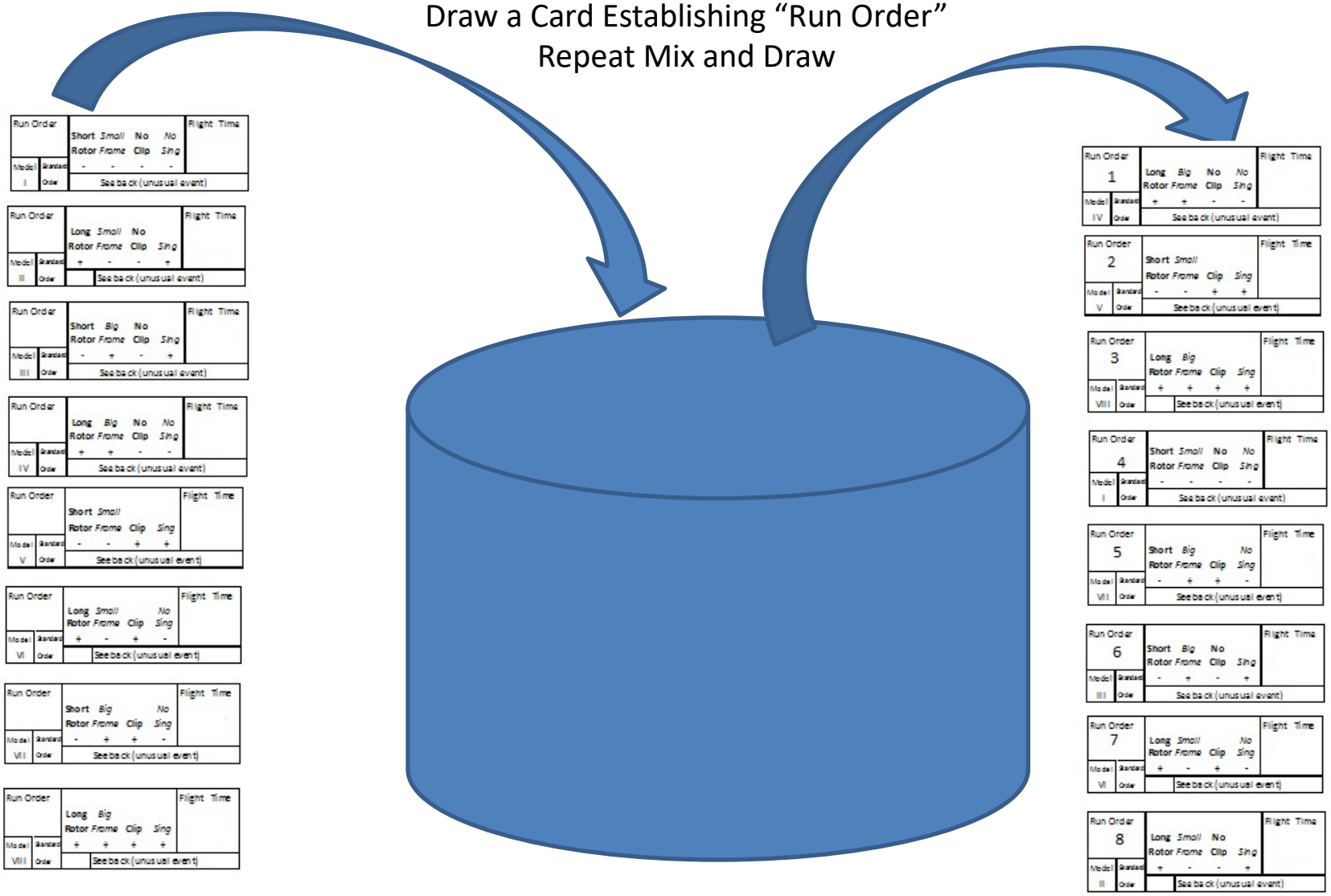
One way to randomize (Accesses the Psychomotor Domain):

Put Each Recipe Card in a Container

Mix

Draw a Card Establishing “Run Order”

Repeat Mix and Draw

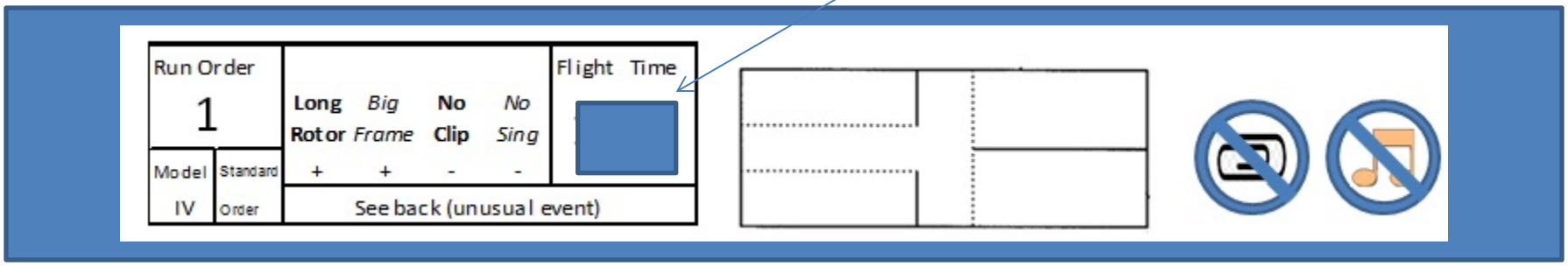


Run #1


1.85

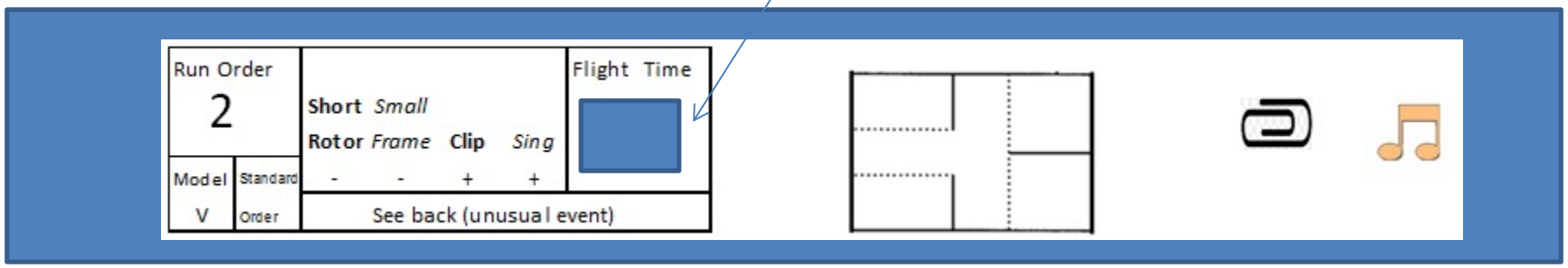

1.79


1.81

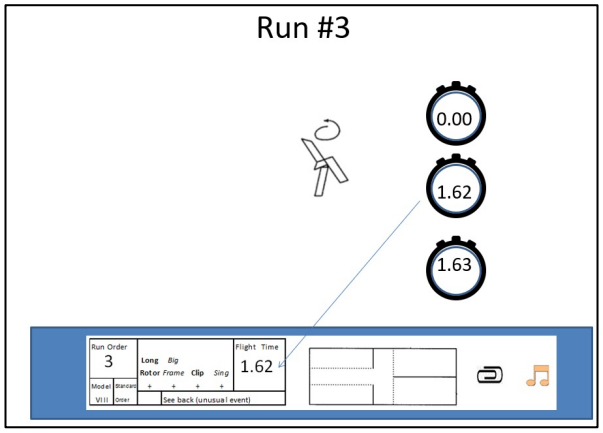

Run Order
1
Model
IV
Standard
Order
Long
Rotor
Big
Frame
No
Clip
No
Sing
+
+
-
-
Flight Time
See back (unusual event)

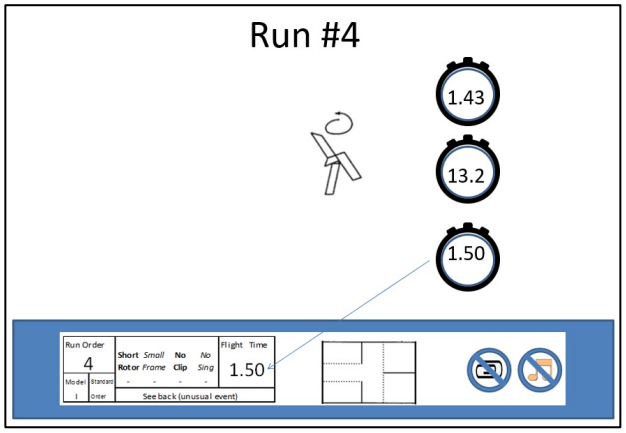
Run #2

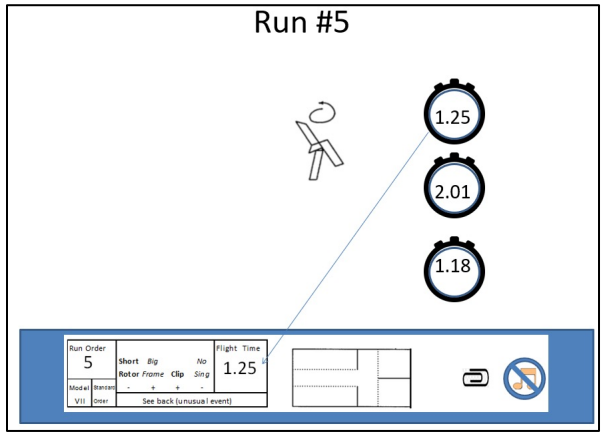

1.43
0.00
1.78

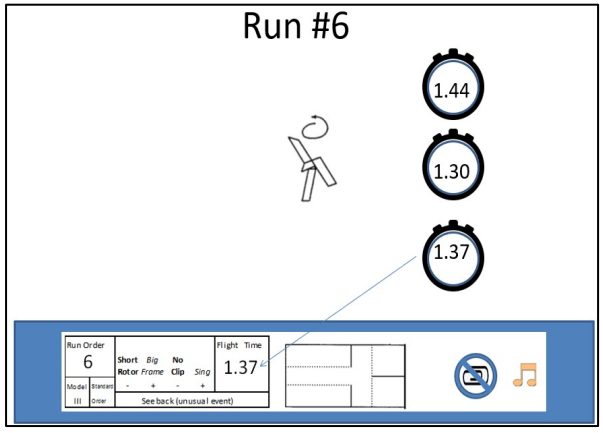

Run Order
2
Model
V
Standard
Order
Short Small
Rotor Frame Clip Sing
- - + +
See back (unusual event)
Flight Time

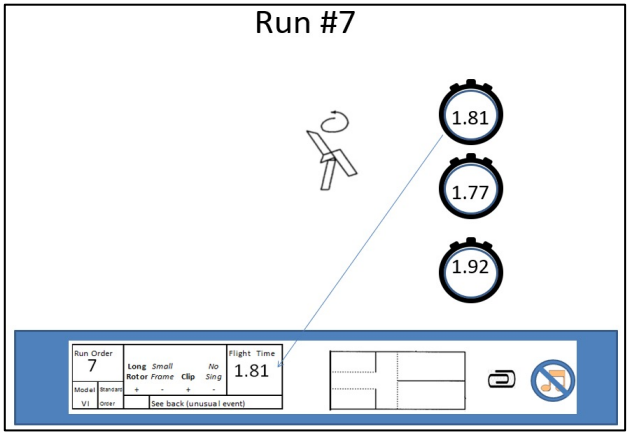
Repeat for Runs 3-8
Remember: Each Is a Different Helicopter Design

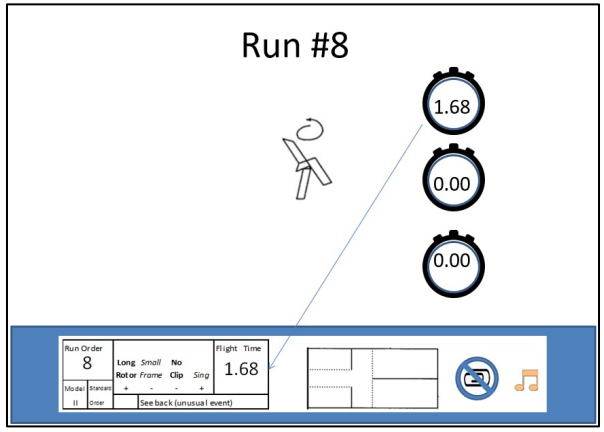




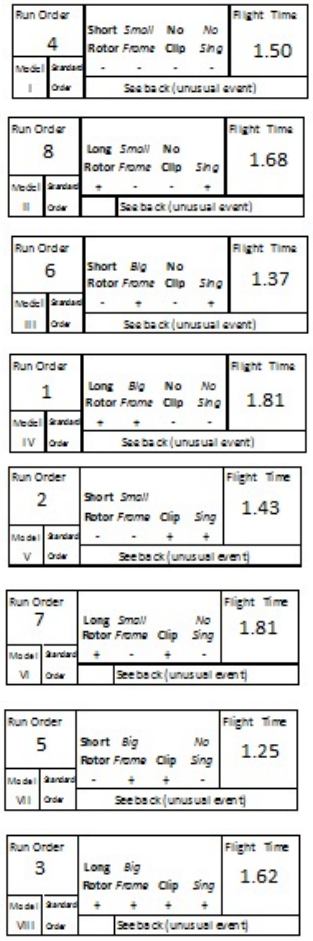








Put “Recipe Cards” In Standard Order
Calculate the Grand Average



Run Order	Rotor	Frame	Clip	Sing	Flight Time
4	Short	Small	No	No	1.50
Model / Standard Order	-	-	-	-	
I	See back (unusual event)				
8	Long	Small	No	Sing	1.68
II	+	-	-	+	
	See back (unusual event)				
6	Short	Big	No	Sing	1.37
III	-	+	-	+	
	See back (unusual event)				
1	Long	Big	No	No	1.81
IV	+	+	-	-	
	See back (unusual event)				
2	Short	Small			1.43
V	-	-	+	+	
	See back (unusual event)				
7	Long	Small		No	1.81
VI	+	-	+	-	
	See back (unusual event)				
5	Short	Big		No	1.25
VII	-	+	+	-	
	See back (unusual event)				
3	Long	Big			1.62
VIII	+	+	+	+	
	See back (unusual event)				

Total 12.47

Average
(Total/8) 1.155875

Next, Group By Rotor Length

Run Order	Rotor	Frame	Clip	Sing	Flight Time
4	Short	Small	No	No	1.50
Model Standard Order	-	-	-	-	
I	See back (unusual event)				

Run Order	Rotor	Frame	Clip	Sing	Flight Time
8	Long	Small	No		1.68
Model Standard Order	+	-	-	+	
II	See back (unusual event)				

Run Order	Rotor	Frame	Clip	Sing	Flight Time
6	Short	Big	No		1.37
Model Standard Order	-	+	-	+	
III	See back (unusual event)				

Run Order	Rotor	Frame	Clip	Sing	Flight Time
1	Long	Big	No	No	1.81
Model Standard Order	+	+	-	-	
IV	See back (unusual event)				

Run Order	Rotor	Frame	Clip	Sing	Flight Time
2	Short	Small			1.43
Model Standard Order	-	-	+	+	
V	See back (unusual event)				

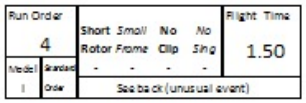
Run Order	Rotor	Frame	Clip	Sing	Flight Time
7	Long	Small		No	1.81
Model Standard Order	+	-	+	-	
VI	See back (unusual event)				

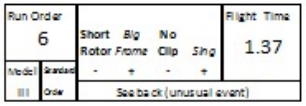
Run Order	Rotor	Frame	Clip	Sing	Flight Time
5	Short	Big		No	1.25
Model Standard Order	-	+	+	-	
VII	See back (unusual event)				

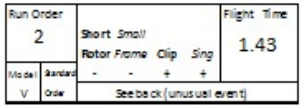
Run Order	Rotor	Frame	Clip	Sing	Flight Time
3	Long	Big			1.62
Model Standard Order	+	+	+	+	
VIII	See back (unusual event)				

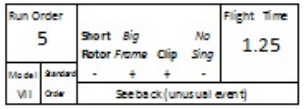
CALCULATE MAIN EFFECT OF DIFFERENT ROTOR LENGTHS

Short Rotor

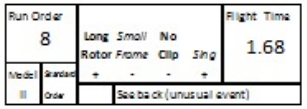


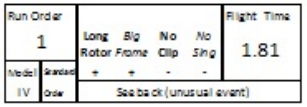


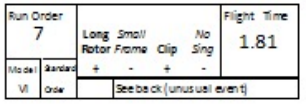


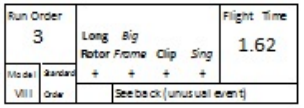


Long Rotor



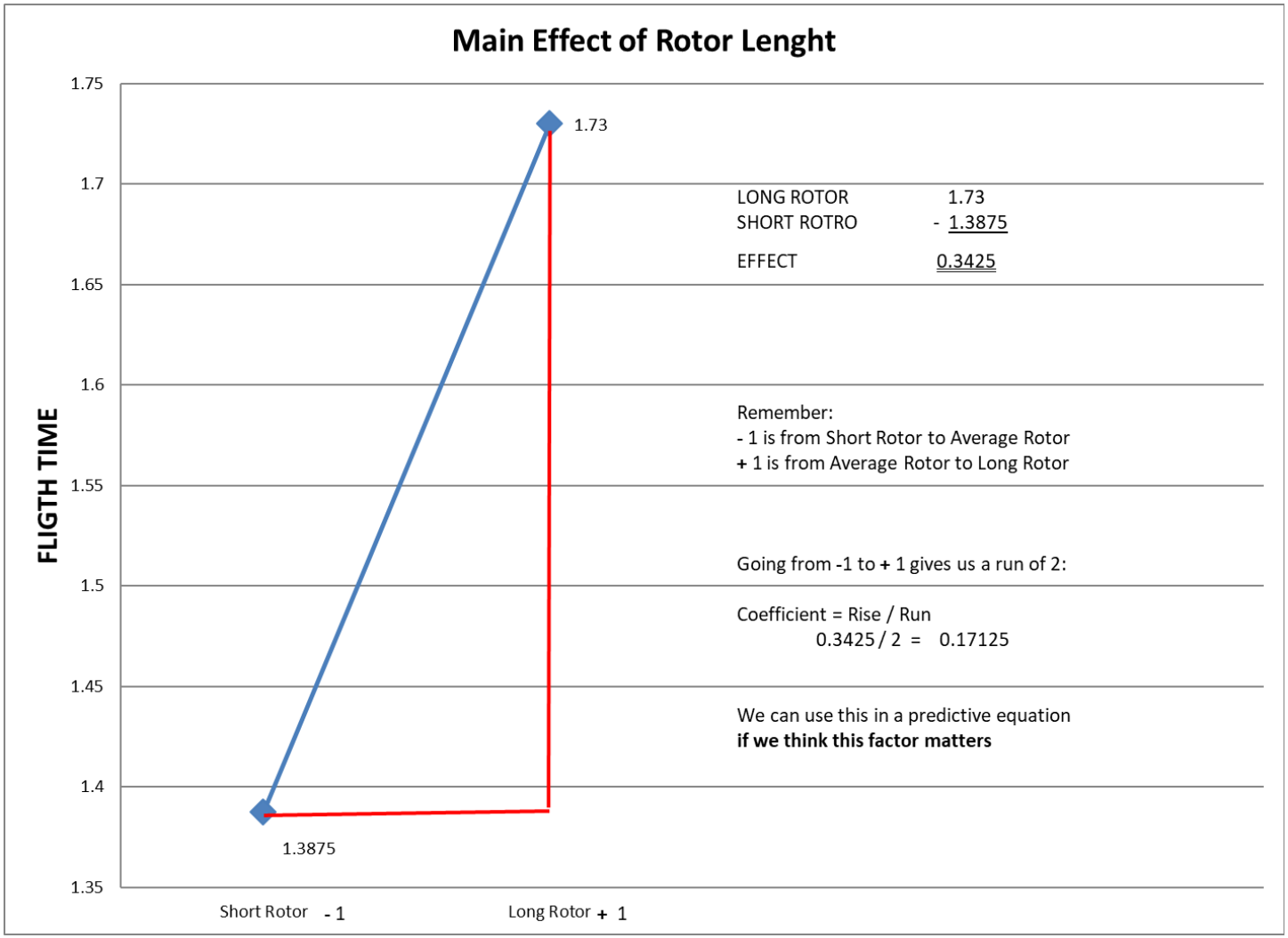






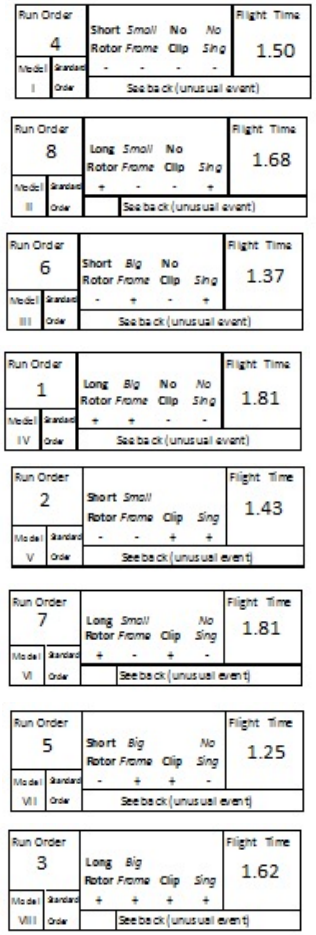
Total	5.55		6.92
Avg. (Total / 4)	1.3875		1.730
Effect (Long – Short)		0.3425	
Coefficient (Effect / 2)		0.17125	

Why We Divide The Effect by 2 (*in this example)* To Obtain Equation Coefficients



CALCULATE MAIN EFFECT OF DIFFERENT FRAME SIZES

Small Frame

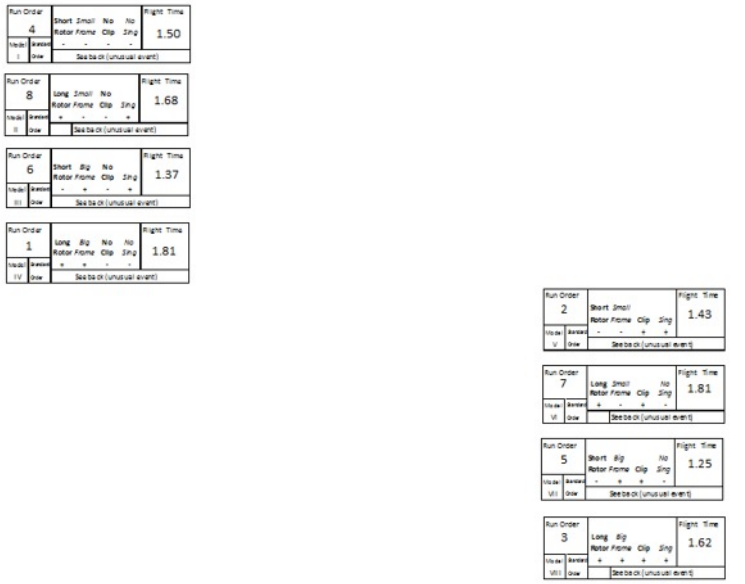


Big Frame

Total	6.42	6.05
Avg. (Total / 4)	1.605	1.513
Effect (Long Avg. – Short Avg.)	-0.0925	
Coefficient (Effect / 2)	-0.04625	

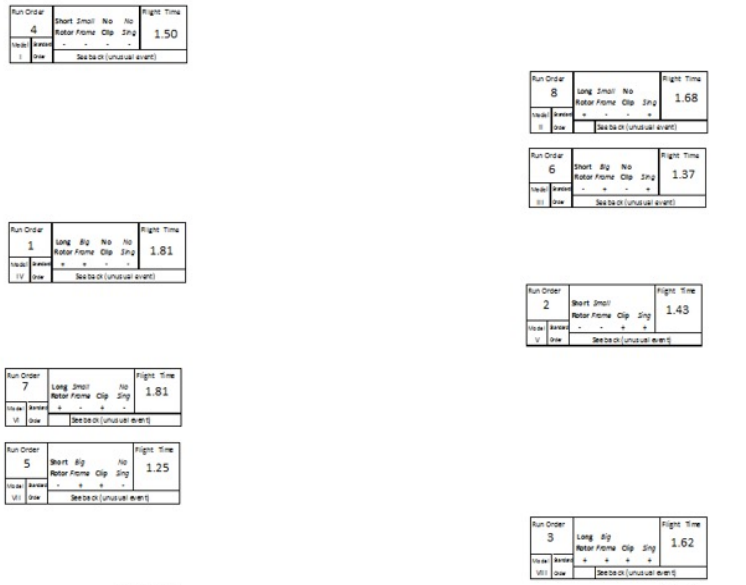
Calculate Other Two Main Effects

CALCULATE THE MAIN EFFECT OF A PAPER CLIP

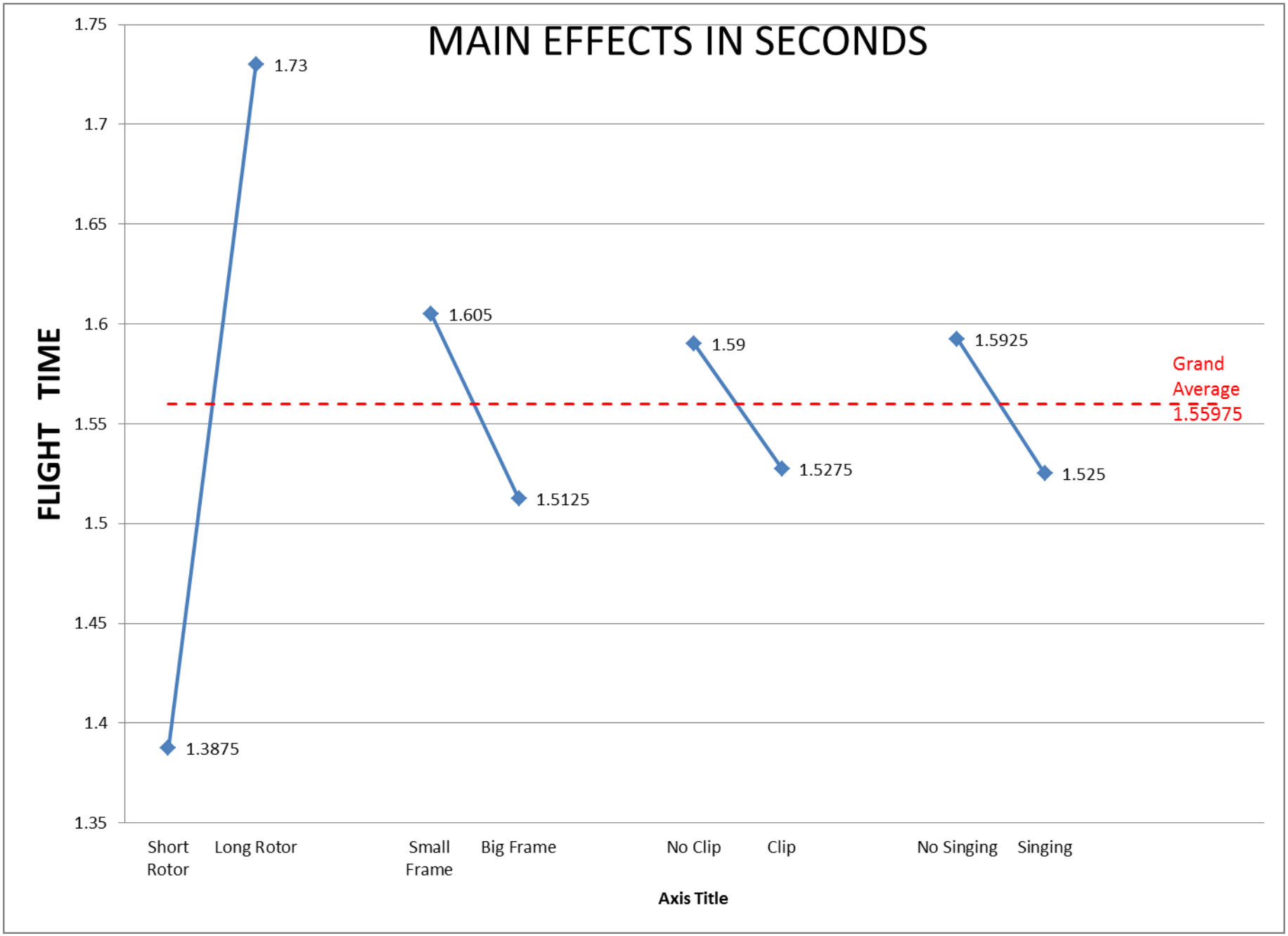


Total	6.36	6.11
Avg, Total / 4)	1.590	1.528
Effect (Long Avg. – Short Avg.)	-0.0625	
Coefficient (Effect / 2)	-0.03125	

CALCULATE THE MAIN EFFECT OF SINGING



Total	6.437	6.1
Avg. (Total / 4)	1.5925	1.525
Effect (Long Avg. – Short Avg.)	-0.0675	
Coefficient (Effect / 2)	-0.03375	


MAIN EFFECTS IN SECONDS
FLIGHT TIME
1.75
1.7
1.65
1.6
1.55
1.5
1.45
1.4
1.35
1.73
1.3875
1.605
1.5125
1.59
1.5275
1.5925
1.525
Grand Average 1.55975
Short Rotor
Long Rotor
Small Frame
Big Frame
No Clip
Clip
No Singing
Singing
Axis Title

The Helicopter Experiment

- "The Helicopter Experiment" is used extensively in 6-Sigma/Quality/Statistics training in industry
- More Sophisticated Versions:
  - Key Source:
    - *George Box, FRS:*
      - Teaching Engineers Experimental Design With A Paper Helicopter, *Quality Engineering,* 1992, Vol. 4, No. 3, pp. 453-459
      - Also Report No. 76 of the Center for Quality and Productivity Improvement of the University of Wisconsin
  - Slightly more complex Paper Helicopter Experiment:
    - *Matthew Barsalou:*
      - Teaching DoE with Paper Helicopters and Minitab http://www.minitab.com/en-us/Published-Articles/Teaching-DoE-with-Paper-Helicopters-and-Minitab/
  - More Advance Paper Helicopter Experiment
    - *Erik Barry Erhardt*
      - Designing a Better Paper Helicopter Using Response Surface Methodology. *Stats*, Issue 48, pp. 14-19, also see p. 2.
  - Statistical Software:
    - DesignExpert https://www.statease.com/
    - JMP https://www.jmp.com/en_us/applications/design-of-experiments.html
    - Minitab http://www.minitab.com/en-us/

More Information

- Box, G. E. P. (1992), "Teaching Engineers Experimental Design with a Paper Helicopter," *Quality Engineering*, 4, 453–459. Found at Teaching Engineers Experimental Design with a Paper Helicopter (williamghunter.net) also see Learning Design of Experiments with Paper Helicopters » Curious Cat Science and Engineering Blog (curiouscatblog.net)
- and for a more advanced optimization approach see Erik Barry Erhardt "Designing a Better Paper Helicopter Using Response Surface Methodology," *Stats* Issue 48, pp. 14-21.
- 2004-08Stat-Teaser.qxd (statease.com)
- Creativity defeats sensibility for paper helicopter fly-off « Stats Made Easy
  - Video of paper-helicopter fly-offs at South Dakota School of Mines & Technology « Stats Made Easy
- Learning Design of Experiments with Paper Helicopters and Minitab

Discussion

- **Are You Interested In Continuing the Conversation?**
  - fluharty.earlydoe@gmail.com

- **Should Introductory Statistics Students See a Demonstration—or Preferably Participate in the Planning, Execution, and Analysis—of a Simplified Multifactor Statistically Design of Experiments (DoE)?**
- **What About Students Who Do Not Take Stat 101?**
- **Is the Recipe Card Approach A Good Example of How To Introduce DoE?**
  - **How Can It Be Improved?**

Why Introduce DoE in Introductory Statistics?

- Why (Educational/Cognitive):
  - Multi Causal Systems (Understanding Needed in Science and Society)
  - Process of Scientific Discovery
    - Iterative
    - Discovery Vs. Demonstrative
    - Discuss Engineering Trade-Offs
  - Omnipresence of Variation
  - Interactions, Predictive Equations, and Optimization
  - Randomization
  - Statistical vs. Practical Significance (if Use Software)
  - Efficiency vs. One-Factor-At-A Time
  - Use of Statistical Graphics
  - Used Business and Industry
    - Manufacturing (Six Sigma Programs)
    - Marketing Experiments
- Pedagogy/Andragogy
  - Cognitive Domain
  - Psychomotor Domain
  - Affective Domain
    - For the Paper Helicopter Example, Model of Autorotation (Flight Time) if Engine Fails