Students' Images of Randomness Post-Instruction

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Introduction

Randomness is an idea that underpins much of Statistics; from probability to randomization for experiments. Given randomness's central role, how students think about randomness is critical to the field of Statistics Education. While there has been research on individuals' understanding of randomness (e.g., Falk 1991, Falk and Konold 1994, Kaplan, Rogness, and Fisher 2014) centered within a classroom, there is little research on students' understandings post instruction. This poster shares the results of three interviews with undergraduate students and their understandings of randomness, one year after instruction. Two students identified that there were two distinct usages of randomness (one everyday, one technical). While no student discussed a fully productive meaning for randomness, Colin came close.

Methods

The present study is part of a larger study investigating students' understandings of stochastic processes and took place at a large public university located in the desert Southwest of the United States. All students had successfully completed an introductory statistics course the year prior to the clinical interviews.

- Question 1: How would you explain randomness?"
- Question 2: Select all of the following situations that you believe match your meaning for "random".

A. Tom and Harry are in the break room discussing what they thought about Star Wars: The Force Awakens. While describing what he liked about the movie, Tom said "Oh, did you know that Linda (a co-worker) is Lutheran?" Harry replied, "That's random." [Left-field]
B. You're at home, someone knocks on your door and you don't know who it is. [Unknown/Unpredictable]
C. You and your two closest friends are trying to resolve who gets to choose what movie to see. One friend doesn't care but the other one and you both want to go see different movies. The neutral friend picks a number at random and the closest of the other two friends wins. [Left-field, Unknown/Unpredictable]
D. Nothing is ever random; there is always a reason that things occur. [Ordained]
E. Everything is random. [Chaos]
F. A sequence is random when you can't find a pattern to it; like the number pi. [Sequence Complexity]
G. A sequence is random when you can't find a pattern, but you can use it predict something in the long-run. [Sequence Attribute]
H. None of these match my meaning for "random".

Bonnie

Bonnie, an elementary education major, ended her introductory statistics course with a B+. Bonnie's course followed a typical American statistics curriculum focused on procedures.

Bonnie's conveyed meaning for randomness revolves around her imagining the event having an impact on the situation. If an event affects the situation (e.g., speaking order, nail polish color, what movie friends go see), then Bonnie sees that event as being random.

"Okay. So, for randomness...(long pause) I want to go back to the term 'everything is random' but in reality, it's not I want to say but...random is, I...I believe it would be the outcome, err, how something would affect a long-term run almost, of a situation…just how randomness can affect on a situation. Going back to the example of the movies where by picking a random number, whoever is closest to it, well that can affect the long-run if they want to see a scary or a funny movie. One person could be sitting there being all sad and upset the whole time while the other person enjoying the movie."

Danielle

Danielle is a conservation biology and ecology major and received a B in her introductory statistics course, which was also a typical statistics curriculum (i.e., procedural).

Danielle conveyed two meanings for randomness. The first revolved round an individual knowing why the event happened. If you don't know why, then the event happened by chance and was random. The second of Danielle's conveyed meanings centered around whether or not she could discern a pattern. Central to this second meaning was *her* discernment; a sequence could be random to *her* as long as she couldn't see a pattern, regardless of whether someone else could.

"How would you explain the idea of randomness to another person? No discernible pattern. So, whatever happened or whatever's going on, there's no discernible pattern to whatever it is…Yes, until someone can tell me the pattern. Until I see the pattern, it would be random. So, random acts probably, could probably have a pattern, but until the pattern is revealed, I would consider it random."

Colin

Colin majored in genetics, cell, and developmental biology, getting an A in his introductory statistics course. Unlike Bonnie and Danielle, Colin's introductory statistics class followed a reform curriculum focused on helping students build productive meanings for statistical concepts.

Colin conveyed two meanings for randomness: one that is non sequitur to the person's anticipations for the current situation and the second revolving around a sequence that does not have a term-based pattern when there are sufficiently many terms.

"A sequence is random if you can't find a pattern, but you can use it to predict something in the long run. I say, it's random. Uhhh, cause if you can't find a pattern then you, you can't, yeah, then next value in that sequence you can't predict what that value will be but you can, on the large scale though, in the long-run, you can predict what the values will gravitate towards. So, I would say that's random."

Construct Map

Attribute of a Process (Most Productive)
The student conveys of "randomness" as a property of a process that entails an image of unpredictability in short-run, while anticipating the predictability in the long-run and minimizes sources of bias. A random process will produce a sequence that has 1) no term-based pattern, 2) a sufficiently complex description, and 3) adheres to the Principle of the Impossibility of a Gambling System. Adapted from Kolmogorov (2013), Liu & Thompson (2002), and von Mises (1981).

Sequence Complexity
The student conveys that a list/sequence is "random" if the individual's attempt to describe the list/sequence is to essentially repeat the sequence as given. The individual cannot condense/reduce the list/sequence to a term-based pattern or set of rules that is less complex than the sequence as given. Drawn from Falk & Konold (1994).

Absence of Long-run Pattern
The student conveys that a sequence is random provided that you have a sufficiently large enough number of trials from the generating process to ensure that there is no term-based pattern to the sequence.

Lack of Discernable Pattern
The student conveys that "random events" have a lack of a discernable pattern. Until the pattern becomes clear to the student, she will view the events as random even while acknowledging that to someone who sees that pattern, the events are not random.

Randomness as Chance
The student conveys that a "random event" is an outcome that occurs out of a collection of other possible outcomes, each of which is equiprobable. Drawn from Bennett (1993), Kaplan et al. (2014), and Kuzmak (2016) .

Left-field or Non Sequitur
The student conveys that events such as sudden switches in conversation topic, unanticipated question, and unexpected images as being "random". Inspired by Liu & Thompson (2002).

Random as Unknown Reason
The student conveys that a "random event" is one that happens but the student does not know why. For example, upon hearing unexpected knocking on a closed door, a student with this way of thinking would say that the event is random to her because she does not know why someone is knocking on the door. However, to the person knocking, the event is not random.

Random as Unknown Result
The student conveys that a "random" event is equivalent to not knowing the outcome. For example, upon hearing knocking on a closed door, a student with this way of thinking will say that some "random" person is at the door since he does not who is at the door. Drawn from Saldanha & Thompson (2014).

Impacting the Situation
The student conveys that a "random" outcome impacts a situation in a way that an alternative outcome would not. The student imagines that situation as continuing on from what is presented and can incorporate affective consequences in her determination of whether the outcome impacts the situation.

Ordained
The student conveys that "random" events result from a chain of events meant to occur. Thus, the student believes that nothing is random.

Chaos (Least Productive)
The student conveys that all events are random and that whatever happens is the result of happenstance.

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