INVESTIGATING FUTURE SECONDARY MATHEMATICS TEACHERS' DEVELOPMENT IN STATISTICS

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Changing Expectations for Teaching K-12 Statistics

- *Statistics Common Core Content Standards*
  - Incorporated ***throughout*** the 6-12 mathematics curriculum
  - Intended to be presented at a more ***intuitive*** level
  - A ***data-analytic and randomization*** approach is recommended
  - Use ***technology*** to explore concepts and analyze real data

Reflect

How do you think secondary preservice teachers (PSTs) would answer these questions?

What did you first learn within the subject of statistics and **how** was it taught to you?

Now, thinking as a teacher, **what** (if anything) would you do differently if you were teaching what you first learned and **why**?

PSTs' Responses

What did you first learn within the subject of statistics and **how** was it taught to you?

- **When?**
  - Middle School (4 out of 12)
  - High School (3 out of 12)
  - College (5 out of 12)

- **What?**
  - Descriptive statistics
  - Vocabulary (mean, median, mode)
  - Prep for AP Exam

- **How?**
  - "We were taught a lot of fill in the blanks and a very step driven process."
  - "This information [vocab] was taught through lecture and was part of the memorization requirement on the first test."
  - "We were taught what they [vocab] meant, but not why they were used.

PSTs' Responses

Now, thinking as a teacher, **what** (if anything) would you do differently if you were teaching what you first learned and **why**?

- "I would like to think I could teach students how to do it without the fill in the blanks … Unfortunately, if you are trying to learn it for an AP test, …, I think I would rather give the kids some of the fill in the blanks so that they know how to do each and when they are comfortable with it, have them explore the reasons behind why."

- "Since students learn by doing better than lecture, I would … let students start doing activities. The knowledge that students need to know can be taught when students realize they need it."

- "I don't think I would pound students with terms and definitions right away. Instead, I would explore various distributions and data sets and allow students to see the terms in action as they learn the definitions."

PSTs' Responses

Now, thinking as a teacher, **what** (if anything) would you do differently if you were teaching what you first learned and **why**?

- "I would make sure statistics was not taught at the very end of the year as a separate subject to explore if we have time. I would also help students see why we care about the mean and the median, when one is more accurate than the other, and go beyond the math of it. Math and statistics are two separate entities, and should be taught as such, but we should also explore how they sometimes intervene with the other."

Secondary Teacher Education Preparation Programs

- PSTs are not prepared to teach statistics as intended.
  - *"Things such as margin of error are just phrases I remember hearing."*
- "Teacher education programs must adapt to meet the challenges of preparing preservice secondary (Grades 6-12) mathematics teachers to teach all aspects of mathematics included in the CCSSM." (Lovett & Lee, 2017)

Motivating Question

How can we better prepare our secondary pre-service mathematics teachers to teach statistics?

Statistics Requirements of our Preparation Program

PSTs typically take the following courses during their sophomore year:

1. **Stat 216: Introduction to Statistics**
   - *Simulation-based curriculum (Tintle et al., 2016)*
   - *TEAL classroom (flipped classroom)*

2. **Stat 217: Intermediate Statistical Concepts**
   - *Introduction to R*
   - *Inference-based*
   - *More lecture-based*

Study Context and Setting

Where else can we incorporate statistics?

- **Mathematical Modeling for Teachers**
  - *3-credit senior capstone course for PSTs taken the semester before student teaching*
  - *Focus: mathematical and statistical models*
  - *Guest lecturers included statisticians and statistics education researchers*

- **Participants**
  - *12 secondary pre-service mathematics teachers*
  - *Varied in their statistical knowledge and experience*
  - *Most concurrently enrolled in High School Methods course*

Data Collected

- Pre- and post-SETS (*Self Efficacy to Teach Statistics*) high school survey
  - *Harrell-Williams, L.M., Sorto, M.A., Pierce, R.L., Lesser, L., & Murphy, T.J. (2012)*
- Pre- and post-LOCUS (*Levels of Conceptual Understanding in Statistics*) test
  - *National Science Foundation Grant No. DRL-1118168.*
- Statistics tasks completed throughout the semester
  - *Includes core statistics assignments, midterm, final exam, reflections, modeling projects*

Core Statistics Assignments

1. **Squirrel Population** – *Statistical vs. Mathematical Models*
2. **Statistical Inference** – *Where's the Math in Statistics?*
3. **Old Faithful** – *Variability*
4. **100 real or 100 fake coin flips?** – *Introduction to Simulation*
5. **Cars on a Highway** – *Developing a Margin of Error through Simulation*

1. Squirrel Population – *Statistical vs. Mathematical Models*

Part 1

In a city park, the grounds keeper notices that the park's squirrel population is on the decline. The keeper has recorded that the annual growth rate for this population of squirrels is g = -0.50. If the initial population of squirrels is 100, determine the population size at the end of five years.

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MATHEMATICAL MODEL

All obtained the same answer!

Model = Structure

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Part 2

In a city park, the grounds keeper notices that the park's squirrel population is on the decline. The keeper has collected some data and states that each year a squirrel has a 50% chance of survival. If the initial population of squirrels is 100, estimate the population size at the end of five years.

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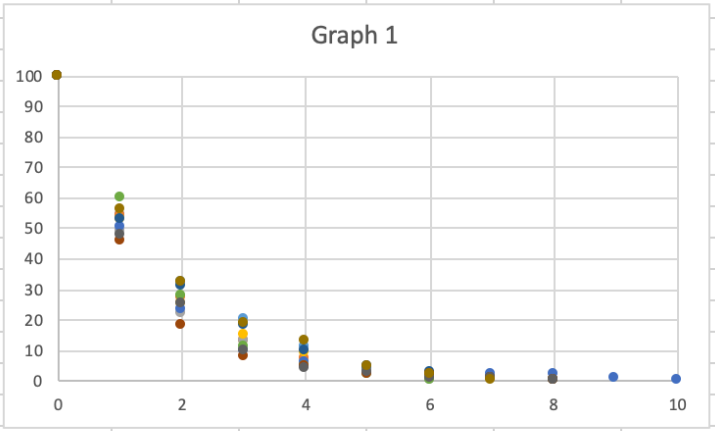
STATISTICAL MODEL

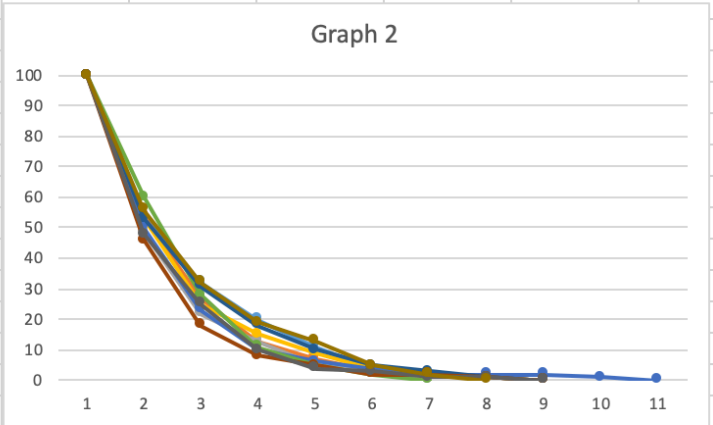
Obtained different answers

Model = Structure + Variability

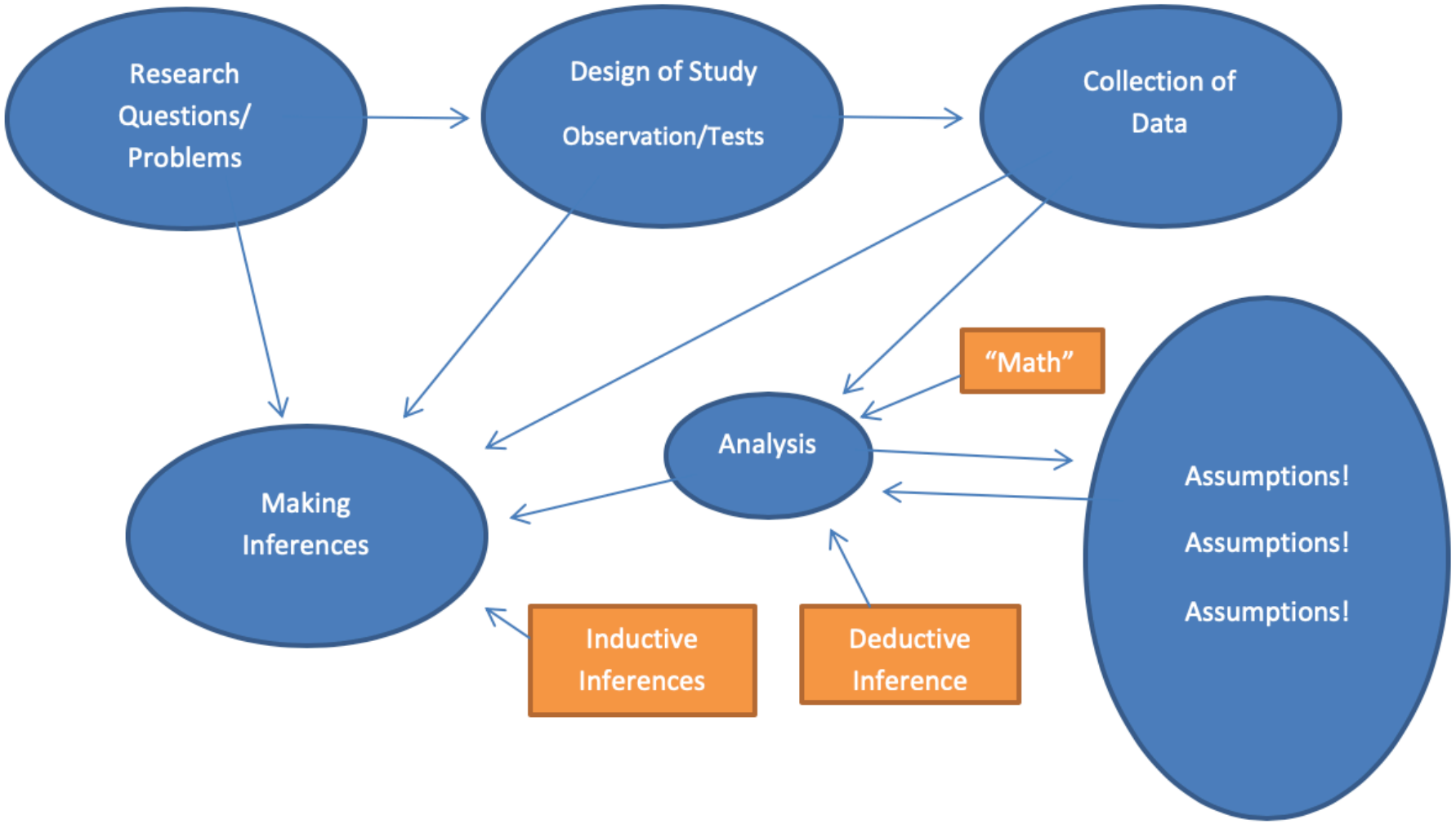
1. Squirrel Population – *Statistical vs. Mathematical Models*

	A	B	C	D	E	F	G	H	I	J	K
1	Year	Rep 1	Rep 2	Rep 3	Rep 4	Rep 5	Rep 6	Rep 7	Rep 8	Rep 9	Rep 10
2	**0**	100	100	100	100	100	100	100	100	100	100
3	**1**	54	54	49	53	50	60	53	46	48	56
4	**2**	32	27	22	25	23	28	31	18	25	32
5	**3**	20	13	13	15	10	11	18	8	10	19
6	**4**	11	7	5	9	6	5	10	5	4	13
7	**5**	4	3	3	4	4	2	5	2	3	5
8	**6**	1	2	0	2	2	0	3	2	1	2
9	**7**	1	1		1	2		1	1	1	0
10	**8**	0	0		0	2		0	0	0	
11	**9**					1					
12	**10**					0					
13											
14	**Discussion:**										
15	Graphs we discussed and made are displayed below. Continue to think about what type of plots are appropriate. We discussed using different plotting characters (stars, boxes, etc.)										
16	and whether or not lines connecting the dots are appropriate. Graph 2's x-axis starts at 1 when we want it to start at 0 - how can we change that.										
17	Keep thinking about these ideas. We'll continue to work on them throughout the semester!										





2. Statistical Inference



3. Old Faithful– *Variability*

- **Data:**

  Wait times (minutes) between successive eruptions

- **Prompt:**

  Hundreds of thousands of people come to Yellowstone Park every year, and most of them include a visit to the Old Faithful geyser, and the question always arises about how long they will have to wait until Old Faithful erupts again. Make a conjecture about the time that someone might expect to wait for Old Faithful to erupt.

- **Additions:**

  Look at K-12 student work and discuss

4. 100 real or 100 fake coin flips? –

Introduction to Simulation

Investigation:

Suppose someone gave you the following sequence of heads ('H') and tails ('T'):

H H T T H T H T H T T T H H T T T H H H T H T T T H H H T H T H T H H H T T T T H
H H H T T T H T H H H T T H H T T T T H H H T H T H T T T T H T H T T H H H H T T
H H H T T T H H T H T H T H T T T T H

Do you think this represents 100 'real' or 100 'fake' flips? Why? How could we decide? *Think about this from the perspective of both a teacher and a student.*

Investigate PSTs' Solutions

- What understanding or knowledge are PSTs demonstrating based on their responses?
  - *As a student?*
  - *As a teacher?*
- What questions would you ask each PST and why?

5. Cars on a Highway – *Developing a Margin of Error through Simulation*

*HSS.IC.B.4 – Use data from a sample survey to estimate a population mean or proportion; **develop a margin of error through the use of simulation models for random sampling.***

Problem:
Describe how to develop a margin of error through the use of simulation models for random sampling. Use the scenario of the speeds of cars on a highway. Include in your description

- How many would you sample?
- What values would you compute?
- How many times would you repeat this sampling?
- How would you display the results of your sampling?
- How would you use your sampling distributions to find a margin of error?
- What inference you can make from this study?
- How would you illustrate that the choices you make as the one designing the random sampling affect the margin of error?
- How this simulation helps build a student's understanding of margin of error.

Investigate PSTs' Solutions

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  - *As a student?*
  - *As a teacher?*
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PSTs' End-of-Year Reflections

- **Statistics is important to teach and is different from mathematics**
  - *In my eyes, Statistics is one of the most relevant courses to a teenager or high schooler. It teaches you how to become a better critical consumer. The students need to know that statistics is so much more than just a number …*

- **Simulation is a powerful instructional tool**
  - *Simulation helps build a student's understanding of margin of error by taking the formula away. Before this class I was taught margin of error with a formula. It didn't make a lot of sense and I didn't get a chance to reason with why it was true. Teaching margin of error with simulation allows a student to know exactly where it comes from and how it makes sense. They self-discover it in a sense. This is more meaningful to them and will stick in their memory better in the long-run.*

PSTs' End-of-Year Reflections

- PSTs more confident about teaching with simulation

	Not at all confident					Completely confident
42. Develop a margin of error for an estimate of a population mean or proportion using simulation models.	1	2	3	4	5	6

Pre-SETS	1	3	2	4	4	3	2	1	1	2
Post-SETS	5	5	6	5	6	6	2	6	5	4

"We CRUSHED margin of error and I feel that I could teach it."

Discussion

- **Ideal …** PSTs have opportunities to develop statistical knowledge for teaching in many classes.
- **Reality** … Most PSTs take at most a general intro stats course.
- **Question** … How do we bridge this gap?

Applications to Teaching

4. Consider the following problem.

> A survey of 625 randomly selected students was conducted to determine the average amount of time students sleep during a weekday. The survey reported an average of 6.5 hours. The survey estimate had a margin of error of half an hour. A margin of error is reported because
>
> a. Sample means vary from sample to sample.
> b. Students may intentionally respond incorrectly.
> c. Students may misunderstand the survey questions.
> d. The people doing the survey may have recorded results incorrectly.

Each of the four choices are common responses to the question of why a margin of error is reported. For each choice, explain why a student would select that answer. What understanding or misunderstanding about margin of error do they have?

Thank you!

Questions?

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